SUBCOURSE EN0591

EDITION C

US ARMY ENGINEER CENTER AND SCHOOL

SURVEYING I

"LET US TRY"

THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT

ARMY CORRESPONDENCE COURSE PROGRAM

SURVEYING I

Subcourse EN0591

EDITION C

United States Engineer Center and School
Fort Leonard Wood, Missouri 65473

7 Credit Hours

Edition Date: November 1997

SUBCOURSE OVERVIEW

This subcourse is designed to give soldiers a practical knowledge of surveying fundamentals and equipment, as well as a review of some of the mathematics needed in surveying operations.

The prerequisite for this course is a basic knowledge of mathematic principles, to include multiplication, division, fractions, and decimals or the completion of the Army Correspondence Course Program (ACCP) Missile & Munitions Subcourse (MM) 0099.

This subcourse reflects the doctrine that was current when it was prepared. In your own work situation, always refer to the latest publication.

At Appendix D, you will find an English/metric conversion chart.

Unless otherwise stated, the masculine gender of singular pronouns is used to refer to both men and women.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn how to use math, geometry, and trigonometry in surveying calculations and be able to identify and understand the uses of surveying equipment.

CONDITION: You will be given the material in this subcourse and an ACCP examination response sheet.

STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70 percent on the subcourse examination.

TABLE OF CONTENTS

Page

GRADING AND CERTIFICATION INSTRUCTIONS

Examination: This subcourse contains a multiple-choice examination covering the material in the five lessons. After studying the lessons and working through the practice exercises, complete the examination. Mark your answers in the subcourse booklet, then transfer them to the ACCP examination response sheet. Completely black out the lettered oval that corresponds to your selection (A, B, C, or D). Use a number 2 pencil to mark your responses. When you complete the ACCP examination response sheet, mail it in the preaddressed envelope provided with the subcourse. You will receive an examination score in the mail. You will receive seven credit hours for successful completion of this examination.

LESSON 1

INTRODUCTION TO SURVEYING

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn the fundamentals of surveying.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn the fundamentals of surveying.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Surveying is a science that deals with determining the relative position of points on or near the earth's surface. These points may be needed for construction to locate or lay out roads, airfields, and structures of all kinds or for cultural, hydrographic, or terrain features for mapping. In the military, these points may be used as target reference points for artillery. The horizontal position of these points is determined from the distances and directions measured in the field. The vertical position is computed from the differences in elevations, which are measured directly or indirectly from an established point of reference or datum.

Surveying is a basic subject in the training of any civil engineer regardless of any ultimate specialization. Its application requires skill as well as knowledge of mathematics (which will be covered later in this correspondence course), physics, drafting and, to some extent, astronomy. This chapter will give you an overview of surveying in general, with emphasis on those areas affecting the duties of a construction surveyor.

Surveying involves fieldwork and office work. Fieldwork consists of taking measurements, collecting engineering data, and testing material. Office work consists of analyzing and computing field data and drawing the necessary sketches.

As your experience increases, you will realize that accuracy in surveying is very important because the results of the surveys are the basis of other factors affecting sound decisions in engineering practice.

PART A - SURVEYING CAREER FIELD

1-1. Career Progression. The normal line of progression for a technical engineering specialist (military occupational specialty (MOS) 51T) and a topographic surveyor (MOS 82D) are discussed in Army Regulation (AR) 611-201.

a. The following listed subcourses for surveyor training are available from the United States (US) Army Engineer School, Fort Leonard Wood, Missouri:

- EN 0591 Surveying I (Mathematics and Surveying Principles).
- EN 0592 Surveying II (Plane-Surveying Operations).
- EN 0593 Surveying III (Topographic and Geodetic Surveying).
- EN 0594 Surveying IV (Construction Surveying).

b. Subcourses EN 0591, EN 0592, and EN 0594 are intended to provide the theoretic training required for a construction surveyor (MOS 51T20). Subcourses EN 0591, EN 0592, and EN 0593 are intended to provide the theoretic training required for a topographic surveyor (MOS 82D20/30).

c. Surveyor training is not complete without some practical work in the field. This should occur after the student has had experience handling and using at least one of each of the more common types of instruments and equipment described in this lesson. Students enrolled in these subcourses should be assigned to an organization or unit that does construction or topographic surveying. If they are not, they must acquire on-the-job training while studying these subcourses.

PART B - GENERAL

1-2. Purpose of a Survey. The earliest application of surveying was for establishing land boundaries. Surveying has also branched out to many fields that parallel the advancement of civil engineering and civilization. Surveyors may be called upon to appear in court to substantiate definite locations of various objects, such as those involved in major traffic accidents, maritime disasters, or even murder cases. Surveying methods may be the same, but their purposes are varied. Generally, surveys are conducted-

- For subdividing or establishing boundaries of land properties. (NOTE: As the value of real estate increases, the demand for good land surveyors also increases.)

- For studying the actual construction of public or private works. (NOTE: As a construction surveyor in the Army, this is the type of surveying you will conduct.)

- On major operations with a higher order of accuracy that only government agencies are equipped to handle; for example, hydrography and the proposed project to tie the world into one triangulation network using earth-orbiting satellites.

Again, although these surveys are for various purposes, the basic operations are the same-they involve measurements and computations or, basically, fieldwork and office work.

1-3. Duties of the Construction Surveyor. In support of construction activities, the surveyor obtains reconnaissance and preliminary data that are necessary at the planning stage. During the construction phase, the surveyor supports the effort as needed. Typical duties of the construction surveyor include-

- Determining distances, areas, and angles.

- Establishing reference points for both horizontal and vertical control.

- Setting stakes or marking lines, grades, and principal points.

- Determining profiles of the ground along given lines (centerlines and/or cross-section lines) to provide data for cuts, fills, and earthwork volumes.

- Laying out structures, culverts, and bridge lines.

- Determining the vertical and horizontal placement of utilities.

1-4. Relative Location and Position. The location of a point on the earth can only be described in terms of the relative location or position of the point with reference to another point. This relative location or position of a point on the earth's surface and the corresponding point on a map may be described in terms of a system of coordinates. Coordinates are quantities that designate the position of a point in relation to a given reference frame. Telling someone that the Main post exchange (PX) is two blocks north of Main Street and three blocks east of Broadway is using coordinates. A point may also be identified by its latitude and longitude and its distance and direction from another point.

a. Coordinates. Coordinates are often used with a grid, which is a network of uniformly spaced straight lines intersecting at right angles. Figure 1-1, page 1-4, shows a grid. The reference frame consists of horizontal and vertical baselines. Each baseline in this example is divided into units of measurement, and each unit is further divided into tenths. The direction of the vertical baseline is called north, and the direction of the horizontal baseline is called east. The intersection of the baselines is called the origin and has a coordinate value of zero-zero. The dot is located 2 units plus three-tenths of a unit more, or 2.3 units, above the horizontal baseline. The dot is also 1.1 units east of the vertical baseline. The coordinates of this grid is 2.3 north and 1.1 east.

b. Latitude and Longitude. Latitude and longitude is the position of any point in relation to where the north-south (NS) line (the zero or Greenwich meridian) intersects the east-west (EW) line (the zero parallel or the earth's equator). This location method is known as the geodetic-coordinates method.

c. Distance and Direction. Another location method is distance and direction. In the following examples, the location is given in terms of the point's direction and the distance from the reference point:

- Example 1: A certain point is located 15 miles southwest of the center of Minneapolis.

- Example 2: In Figure 1-2, standing at the corner of the garage, the tree is 45° clockwise from the edge of the driveway and 50 steps away from the garage.

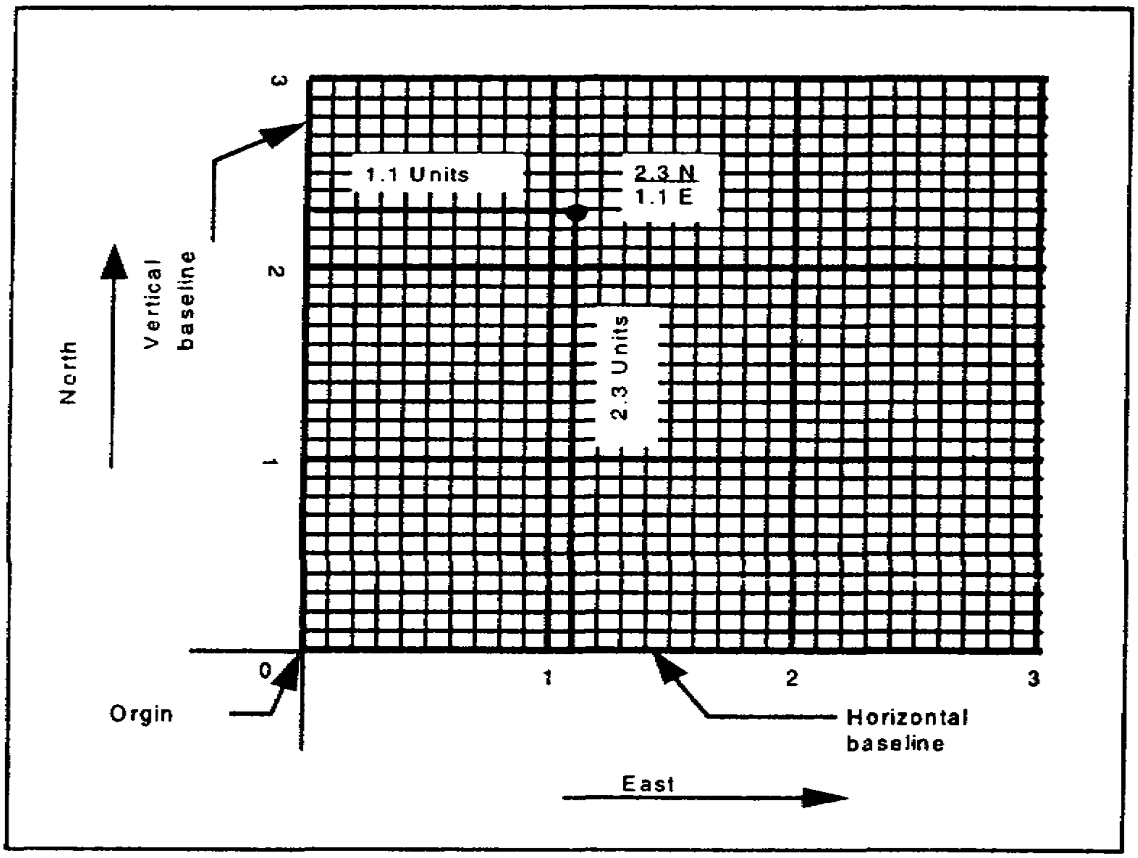

Figure 1-1. Coordinates of a point on a grid

1-5. Plane Surveying. The branch of surveying in which the mean surface of the earth is considered a plane surface is generally referred to as plane surveying. In plane surveying, the earth's curvature is neglected, and computations are made using the formulas of plane geometry and trigonometry. In general, plane surveying is applied to surveys of land areas and boundaries (land surveying) where the areas are of limited extent. Plane surveying is also used when the required accuracy is so low that corrections for the effect of curvature would be negligible as compared to the errors of observations. For small areas, precise results may be obtained with plane-surveying methods, but the accuracy and precision of such results will decrease as the area surveyed increases in size. Generally, plane surveying is done for the location and construction of highways, railroads, canals, and landing strips.

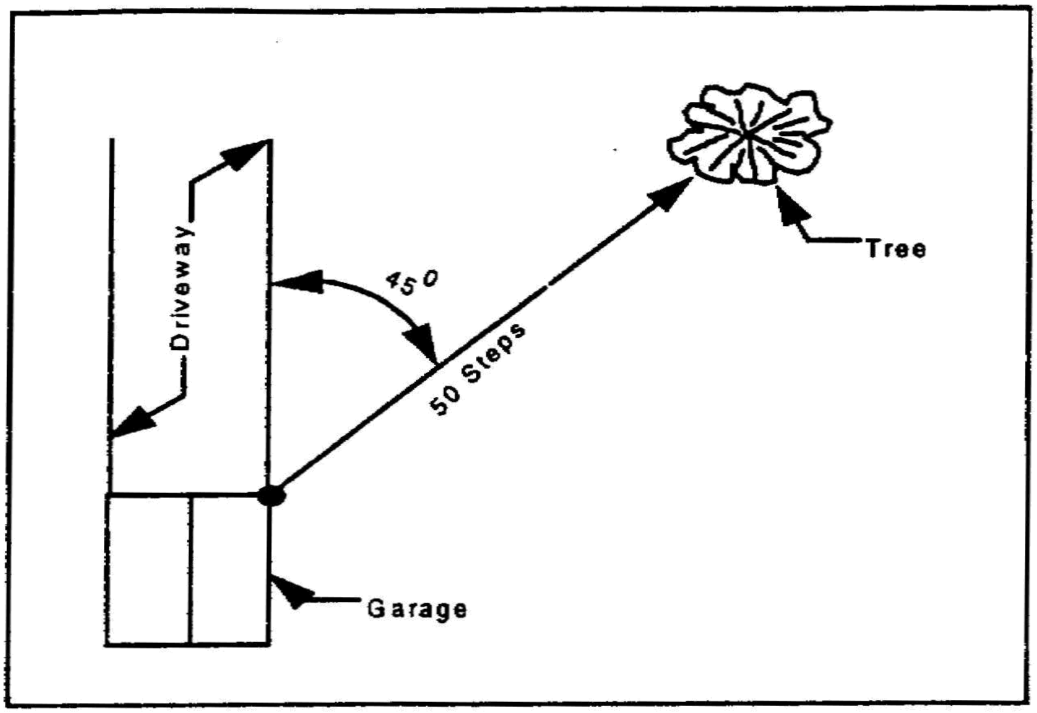

Figure 1-2. Distance and direction

Leveling operations are usually considered a part of plane surveying. The effect of the earth's curvature is automatically taken into account when using standard procedures to determine elevations. Elevations are referred to a certain datum, which is a plane tangent at any point of the earth's surface and normal to the plumb line at that point. This datum is normally taken at an imaginary plane tangent to the surface of mean sea level. Figure 1-3, page 1-6, illustrates the fact that when this datum plane is extended 10 miles out from the point of tangency, the vertical distance (elevation) of the plane above the surface represented by mean sea level is 67 feet, and at a distance of 100 miles, this elevation becomes 6,670 feet. For this reason, the earth's curvature cannot be neglected as a factor in taking even rough elevations. Later, you will learn the importance of maintaining, as much as possible, a balanced distance between the foresight and backsight in leveling operations.

PART C - TERMINOLOGY

1-6. Surveying Terms. The following surveying terms are just a few of the technical terms that you will be using. You will learn most of them in actual practice and through usage.

- *Agonic line*-The line along which the magnetic declination is zero.
- *Angle of inclination*-A vertical angle of elevation or depression.
- *Azimuth*-The angle to a line of sight, measured clockwise from any meridian and range from 0° to 360°.

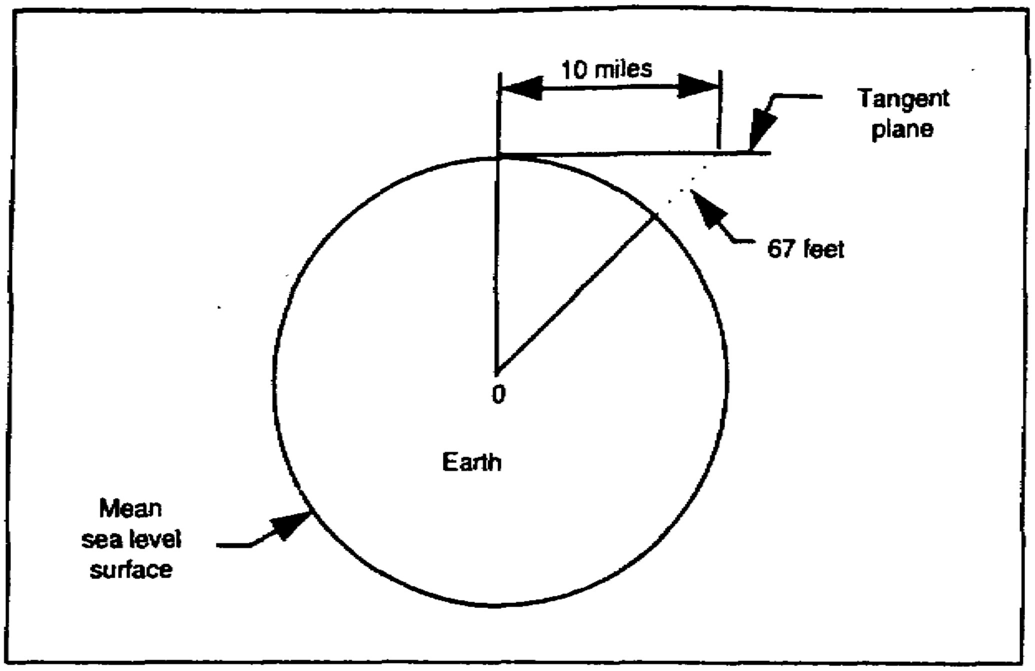

Figure 1-3. The effect of the earth's curvature

- *Blaze*-A mark made on the trunk of a standing tree by chipping off a spot of bark with an ax. It is used to indicate a trail, a boundary, a location for a road, a tree to be cut, and so on.

- *Bubble axis (level vial)*-The horizontal line tangent to the upper surface of the centered bubble, which lies in the vertical plane through the longitudinal axis of the bubble tube.

- *Calibration*-The process of standardizing a measuring instrument by determining the deviation from a standard so as to ascertain the proper correction factors.

- *Collimation*-The act of adjusting the line of sight of a telescopic surveying instrument to its proper position relative to the other parts of the instrument.

- *Collimation line*-The line through the second nodal point of the objective (object glass) of a telescope and the center of the reticle. It is also referred to as the line of sight, sight line, pointing line, and the aiming line of the instrument. The center of the telescope reticle can be defined by the intersection of crosshairs or by the middle point of a fixed vertical wire or a micrometer wire in its mean position. In a leveling instrument, the center of the reticle may be the middle point of a fixed horizontal wire.

- *Contour*-An imaginary level line (constant elevation) on the ground surface; it is called a contour line on a corresponding map.

- *Datum*-Any numeric or geometric quantity that serves as a reference or base for other quantities. It is described by such names as geodetic, leveling, North American, or tidal datum, depending on its purpose when established.

- *Elevation*-The vertical distance of a point above or below an established or assumed point (datum) on a level surface.

- *Geodetic datum*-Datum that forms the basis for the computation of horizontal-control surveys in geodetic surveying. It consists of five quantities: the latitude and the longitude of an initial point, the azimuth of a line from this point, and two constants necessary to define the terrestrial spheroid.

- *Grade (gradient)* -The rate of rise and fall or slope of a line; generally expressed in percent or as a ratio.

- *Horizontal angle*-The angle formed by two intersecting lines on a horizontal plane.

- *Horizontal distance*-A distance measured along a level line. It is commonly thought of as the distance between two points. The distance may be measured by holding a tape horizontally or by measuring the inclined distance between the points. However, the inclined distance is always reduced to its horizontal length.

- *Horizontal line*-A straight line perpendicular to a vertical line at a given point.

- *Horizontal plane*-A plane tangent to a level surface (also called plane of the horizon).

- *Instrument adjustment*-Adjusting the parts of an instrument to obtain the highest practical precision. For example, field adjustments of theodolites include adjusting the bubble tube, circular bubble, line of sight, horizontal axis, telescope bubble tube, vertical circle, and optical plummet.

- *Isogonic line*-An imaginary line or a line on a map joining points on the earth's surface at which the magnetic declination is the same.

- *Legend*-A description, explanation, table of symbols, and so on printed on a map or chart for a better understanding and interpretation of it.

- *Level surface*-A surface that is parallel with the spherical surface of the earth, such as a body of still water.

- *Leveling datum*-A level surface to which elevations are referred. Generally, the adopted datum for leveling in the US is the mean sea level.

- *Measured angles*-Angles that are either vertical or horizontal.

- *Parallax*-An error in sighting that occurs when the objective and/or the crosshairs of a telescope are improperly focused. In testing the focusing of a telescope, the head of the observer must move from side to side or up and down while sighting through the eyepiece. Any apparent movement of the crosshairs in relation to the object image means that parallax is present.

- *Plumb line*-A line (as a cord) that has at one end a weight (as a plumb bob) and is used to determine verticality.

- *Reticle*-A scale on transparent material (as in a telescope) used especially for measuring or aiming.

- *Station*-The location of a definite point on the earth's surface that has been determined by surveying methods. It may be a point on a traverse over which an instrument is set up or a length of 100 feet measured on a given line-broken, straight, or curved.

- *Traverse*-A sequence of lengths and directions of lines between points on the earth, obtained by or from field measurements and used in determining the positions of the points. A traverse may determine the relative positions of the points that it connects in a series.

- *Vertical angle*-An angle between two intersecting lines in a vertical plane. It should be understood that one line lies on the horizontal plane, and the angle originates from the intersection of the two planes.

- *Vertical line*-A line that lies in the vertical plane and is perpendicular to the plane of the horizon, such as the direction of a plumb line.

- *Vertical plane*-A plane that is perpendicular to the horizontal plane.

PART D - FIELDWORK

1-7. Fieldwork Concept. Fieldwork is important in all types of surveys. To be a skilled surveyor, you must spend a certain amount of time in the field to acquire needed experience. The study of this ACCP will enable you to understand the underlying theory of surveying, the instruments and their uses, and the surveying methods. However, a high degree of proficiency in actual surveying, as in other professions, depends largely on the duration, extent, and variety of your actual experience.

a. Project Analysis. The project must be analyzed thoroughly before going into the field. You must know exactly what is to be done, how you will do it, why you prefer a certain approach over other possible solutions, and what instruments and materials you will need to accomplish the project.

b. Speed. You must develop speed in all your fieldwork. This means that you will need practice in handling the instruments, taking observations, keeping field notes, and planning systematic moves. Surveying speed is not the result of hurrying; it is the result of the following:

- The surveyor's skill in handling the instruments.

- The intelligent planning and preparation of the work.

- The process of making only those measurements that are consistent with the accuracy level that is required.

c. Accuracy Levels. Measurements must not be accepted as correct without verification. Verification, as much as possible, must be different from the original measurement method used. The precision of measurement must be consistent with the accuracy level that is required for the surveying type being conducted. The higher the accuracy level, the more time that is required to make the measurement, since greater care and more observations must be taken.

The purpose and type of a survey are the primary factors in determining the accuracy level that is required. This, in turn, will influence the selection of instruments and procedures. First-order triangulation, which becomes the basis or "control" of future surveys, requires a high level of accuracy. At the other extreme, cuts and fills for a highway survey require a much lower accuracy level. In some construction surveys, inaccessible distances must be computed. The distance is computed by means of trigonometry, using the angles and the one distance that can be measured. The measurements must be made to a high degree of precision to maintain accuracy in the computed distance.

d. Maintenance. Fieldwork also includes adjusting instruments and caring for field equipment. Do not attempt to adjust any instrument unless you understand the workings or functions of its parts. Adjusting instruments in the early stage of your career requires close supervision from an experienced surveyor.

e. Factors Affecting Fieldwork. The surveyor must constantly be alert to the different conditions encountered in the field. Physical factors, such as terrain and weather conditions, will affect each field survey in varying degrees. The following are some of these conditions:

- Measurements using telescopes can be stopped by fog or mist.

- Swamps and floodplains under high water can impede taping surveys.

- Sighting over open water or fields of flat, unbroken terrain creates ambiguities in measurements using microwave equipment.

- Bright sunlight reduces light-wave measurements.

However, reconnaissance will generally predetermine these conditions and alert the surveying party to the best method to use and the rate of progress to be expected.

f. Training. The training status of personnel is another factor that affects fieldwork. Experience in handling the instruments being used for a survey can shorten surveying time without introducing errors that would require resurvey.

1-8. Measurements. Fieldwork in surveying consists mainly of taking and recording measurements. The operations are as follows:

- Measuring distances and angles for the purpose of-

- Establishing points and reference lines for locating details (such as boundary lines, roads, buildings, fences, rivers, bridges, and other existing features).

- Staking out or locating roads, buildings, landing strips, and other construction projects.

- Establishing lines parallel or at right angles to other lines.

- Extending straight lines beyond obstacles, such as buildings.

- Performing other duties that may require the use of geometric or trigonometric principles.

• Measuring differences in elevations and determining elevations for the purpose of-

  - Establishing reference points (bench marks).

  - Obtaining terrain elevations along a selected line for plotting profiles and computing grade lines.

  - Staking out grades, cuts, and fills for construction projects.

• Conducting topographic surveys where horizontal and vertical measurements are combined.

• Making soundings in bodies of water for preparing charts for navigation or for developing maps for waterfront structures.

• Recording field notes to provide a permanent record of the fieldwork.

Surveying measurements will be in error to the extent that no measurement is ever exact. Errors are classified as systematic and accidental. Besides errors, surveying measurements are subject to mistakes or blunders. These arise from misunderstanding of the problem, poor judgment, confusion on the part of the surveyor, or simply from an oversight. By working out a systematic procedure, the surveyor will often detect a mistake when some operation seems out of place. The procedure will be an advantage in setting up the equipment, making observations, recording field notes, and making computations.

1-9. Field Notes. The surveyor's field notes must contain a complete record of all measurements made during the survey, with sketches and narration when necessary, to clarify the notes. The best field survey is of little value if the notes are not complete and clear. They are the only record that is left after the field surveying party leaves the site.

a. Field-Note Types. The following are the four basic types of field notes: tabulations, sketches, descriptions, and combinations. The combination method is the most common method because it fits so many overall needs.

(1) Tabulations. Tabulations are the numerical measurements that are recorded in columns according to a prescribed plan. Spaces are also reserved to permit necessary computations. Tabulations, with or without added sketches, can also be supplemented with descriptions.

(2) Sketches. Sketches add much to clarify field notes and should be used liberally when applicable. They may be drawn to an approximate scale, or important details may be exaggerated for clarity. A small ruler or triangle is an aid in making sketches. Measurements should be added directly on the sketch or keyed in some way to the tabular data. A very important requirement of a sketch is legibility. See that the sketch is drawn clearly and large enough to be understandable.

(3) Descriptions. Descriptions may only be one or two words to clarify recorded measurements or may be a lengthy narration if it is to be used at some future time, possibly years later, to locate a surveying monument.

(4) Combinations. Two, or even all three, of the methods can be combined, when necessary, to make a complete record.

b. Field Notebook. A field notebook is a permanently bound book for recording measurements as they are made in the field. Several types are available to record the different kinds of surveying measurements. The front cover of a field notebook should be marked with the name of the project its general location, the types of measurements recorded, the designation of the surveying unit, and other pertinent information as specified by the engineering officer. The inside front cover should contain instructions for the return of the notebook, if lost. The right-hand pages should be reserved for an index of the field notes, a list of party personnel and their duties, a list of the instruments used (plus the dates and the reasons for any instrument being changed during the course of the survey), and a sketch and description of the project. Throughout the remainder of the notebook, the beginning and ending of each day's work should be clearly indicated. When pertinent, the weather, including temperature and wind velocities, should also be recorded. To minimize recording errors, all data entered in the notebook must be checked and initialed by someone other than the recorder.

(1) Legibility. All field notes should be lettered legibly. A mechanical pencil or a number 3 or 4 hard-lead pencil, using sufficient pressure, will ensure a permanent record. Numerals and decimal points should be legible and permit only one interpretation. Notes must be kept in the field notebook and not on scraps of paper for later transcription. Separate surveys should be recorded on separate pages or in different books.

(2) Erasures. Erasures are not permitted in field notebooks. Individual numbers or lines recorded incorrectly are lined out and the correct values added. Pages that are to be rejected are crossed out neatly and referenced to the substituted page. This procedure is mandatory since the field notebook is the book of record and is often used as legal evidence.

(3) Abbreviations. Standard abbreviations, signs, and symbols are used in field notebooks. If there is any doubt as to their meaning, an explanation must be given in the form of notes or legends.

PART E - SURVEYING PARTIES

1-10. Surveying-Party Types. The size of a field surveying party depends on the surveying requirements, the equipment available, the surveying method, and the number of personnel needed for performing the different functions. The following are the surveying parties that will be discussed in this lesson: a leveling party, a transit party, and a stadia party. The other surveying-party organizations generally follow the same pattern, therefore, they will not be discussed.

a. Leveling Party. The smallest leveling party consists of two persons. For differential leveling, one person acts as an instrument man (level man) and the other holds the rod (rod man). Trigonometric leveling requires instrument men to read vertical angles as well. In small parties of this type, the instrument men must record their own notes. Reciprocal leveling can be done by two people but requires separate vehicles for transporting the party and the equipment around the obstruction.

(1) Additional Persons. To improve the efficiency of the different leveling operations, additional personnel are required. A second rod man to alternate on the backsights and foresights will speed up leveling. A recorder will allow the instrument man to take readings as soon as the rod men are in position. For surveys with numerous side shots, extra rod men will eliminate waiting periods while one or two persons move from point to point. In surveys requiring a shaded instrument, an umbrella man can allow the recorder to concentrate on note keeping.

(2) Combined Party. Leveling operations may be run along with a traverse or as part of a taping survey. In these instances, the leveling party may be organized as part of a combined party with personnel assuming duties as required by the workload and as assigned by the party chief.

b. Transit Party. A transit party consists of at least three people: an instrument man, a head chainman, and a party chief. The party chief is usually the note keeper and may double as a rear chainman, or there may be an additional rear chainman. The instrument man operates the transit, the head chainman measures the horizontal distances, and the party chief directs the survey and keeps the notes.

c. Stadia Party. A stadia party should consist of three people: an instrument man, a note keeper, and a rod man. However, two rod men should be used if there are long distances between observed points so that one can proceed to a new point while the other is holding the rod on a point being observed. The note keeper records the data the instrument man calls out and makes the required sketches.

PART F - SURVEYS

1-11. Surveying Types. Generally, surveys are classified by names descriptive of their functions, such as property surveys, mine surveys, hydrographic surveys, and so on. Although surveys are classified by many different names, the methods and instruments used are basically the same. Some of the types of surveys that you might perform as a construction surveyor are discussed below.

a. Land Surveys. A land survey is conducted to establish the exact location, boundaries, or subdivision of a tract of land in a specified area. This type of work is sometimes referred to as cadastral surveying. When this work is primarily done within city limits, it is referred to as city surveying. At present, land surveys generally consist of the following chores:

- Establishing markers or monuments to define and thereby preserve the boundaries of land belonging to an individual, a corporation, or the government.
- Relocating markers or monuments legally established by original surveys. This requires examining previous surveying records and retracing what was done. When some markers or monuments are missing, they are reestablished by following recognized procedures and using whatever information is available.
- Rerunning old land-surveying lines to determine their lengths and directions. As a result of the high cost of land, old lines are remeasured to get more precise measurements.
- Subdividing land into parcels of predetermined sizes and shape.
- Calculating areas, distances, and directions and preparing a land map to portray surveying data so that it can be used as a permanent record.
- Writing a technical description for deeds.

b. Topographic Surveys. A topographic survey is conducted to gather surveying data about natural and man-made land features, as well as elevations. From this information a three-dimensional map may be prepared. The topographic map may be prepared in the office after collecting the field data or done right away in the field by plane table. The work usually consists of the following:

- Establishing horizontal and vertical control that will serve as the framework of the survey.
- Determining the horizontal location and elevation (usually called "side shots") of ground points to provide enough data for plotting when the map is prepared.
- Locating natural and man-made features.
- Computing distances, angles, and elevations.
- Drawing the topographic map.

c. Engineering or Construction Surveys. An engineering or a construction survey is conducted to obtain data for the various phases of construction activity. It includes a reconnaissance survey, a preliminary survey, a location survey, and a layout survey. The objectives of an engineering or a construction survey include-

- Obtaining reconnaissance information and preliminary data that engineers require for selecting suitable routes and sites and for preparing structural designs.
- Defining selected locations by establishing a system of reference points.
- Guiding construction forces by setting stakes or marking lines, grades, and principal points and by giving technical assistance.
- Measuring construction items in place to prepare progress reports.
- Dimensioning structures for preparing as-built plans.

(1) Terms. The American Society of Civil Engineers (ASCE) applies the term engineering surveys to all of the above objectives and construction surveys to the last three objectives only. The Army Corps of Engineers, on the other hand, applies construction surveys to all of the above objectives.

(2) Structures. Engineering and/or construction surveys from part of a series of activities leading to the construction of a man-made structure. The term "structure" is usually confined to something that is built of structural members, such as a building or a bridge. It is used here in a broader sense, however, to include all man-made features, such as graded areas; sewer, power, and water lines; roads and highways; and waterfront structures.

d. Route Surveys. A route survey is conducted for locating and constructing transportation or communication lines that continue across country for some distance, such as highways, railroads, open conduit systems, pipelines, and power lines. Generally, the preliminary survey for this type of work takes the form of a topographic survey. In the final stage, the work may consist of the following:

- Locating a centerline, usually marked by stakes at 100-foot intervals (called stations).
- Determining elevations along and across a centerline for plotting a profile and cross sections.
- Plotting a profile and cross sections and fixing grades.
- Computing the volumes of earthwork and preparing a mass diagram.
- Staking out the extremities for cuts and fills.
- Determining drainage areas to be used for ditches and culverts.
- Laying out structures, such as bridges, culverts, and so on.

- Locating right-of-way boundaries, as well as staking out fence lines, if necessary.

PART G - CONTROL SURVEYS

1-12. Control Types. Control surveys establish reference points and reference lines for detail surveys. Control may be either horizontal or vertical.

a. Horizontal Control. Horizontal control is a basic framework of points in which the horizontal position and interrelationship of have been accurately determined.

(1) Horizontal Control by Traversing. A surveying traverse is a sequence of lengths and directions of lines between points on the earth, obtained by or from field measurements and used in determining the positions of the points. A surveying traverse may determine the relative positions of the points that it connects in a series.

(a) Closed Traverse. A closed traverse is one that ends at the point at which it began (see Figure 1-4, page 1-16).

(b) Open Traverse. An open, or open-end, traverse is one that ends at a point other than the one at which it began (see Figure 1-5, page 1-16).

(2) Horizontal Control by Triangulation. Triangulation is a method of surveying in which the stations are points on the ground that are located in a series of triangles. The angles of the triangulation net are measured by using instruments, and the lengths of the sides are derived by computation from selected sides that are termed baselines-the lengths of which have been obtained from precise direct measurements on the ground.

b. Vertical Control. Vertical control (also called elevation control) is a series of bench marks or other points of known relative vertical position that are established throughout a project. In a topographic survey, for example, a circuit of bench marks is established over an area at convenient intervals (usually every half mile along a coordinate system on government property) to serve as starting and closing points for leveling operations. They also serve as reference marks for grades and finished floor elevations for structures in subsequent construction work. Since these bench marks will be needed from time to time to establish other elevations, it is important that the work be accurately done so that elevations referred to by one bench mark will check with those referred to by any other bench mark in the circuit. The bench marks must be established in a definite point of more or less permanent character so that they will not be disturbed.

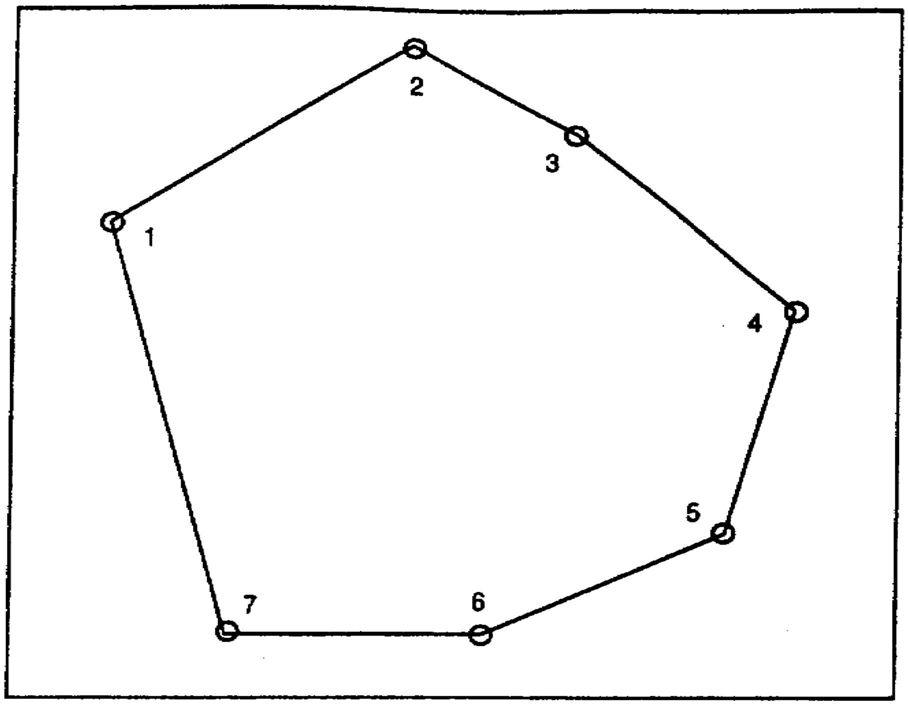

Figure 1-4. Closed traverse

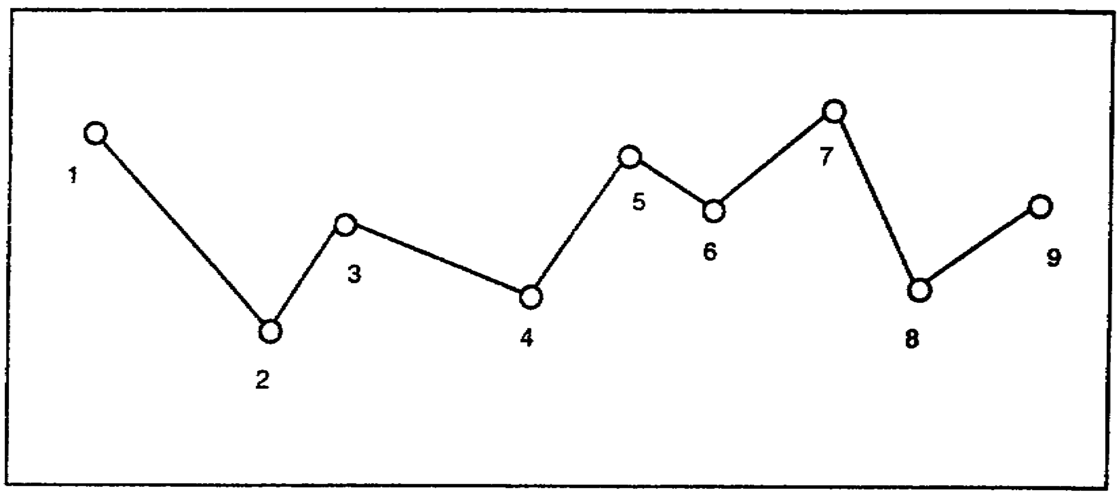

Figure 1-5. Open traverse

PART H - PROFESSIONAL SOCIETIES AND MANUFACTURERS

1-13. Surveyors' Professional Societies. US surveyors have two professional societies: the Surveying and Mapping Division of the ASCE and the American Congress on Surveying and Mapping. You must be a registered civil engineer to become a member of the ASCE or a registered surveyor to become a member of the American Congress on Surveying and Mapping. A working committee of these organizations may be formed to resolve technical problems affecting the art and science of surveying, such as the

Committee of Surveying and Mapping Division, ASCE. The executive committee of these organizations may also appoint representatives to represent them in international conventions, such as the International Geodetic and Geophysical Union.

1-14. Manufacturers of Surveying Instruments and Supplies. Berger, Bruning, Dietzgen, Gurley, Kern, Keuffel & Esser, Lufkin, Post, Litton Systems Inc. and Wild, along with others, are well-known manufacturers of surveying equipment. Manufacturers publish, in pamphlets and booklets of various kinds, a great deal of valuable information on surveying equipment and its use and are usually glad to provide such information, without charge, to anyone requesting it.

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LESSON 1

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. There is only one correct answer to each item. When you complete the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. The location of a point may be determined by its _______________ from another point.

   A. Elevation
   B. Distance and direction
   C. Horizontal angle
   D. Vertical angle

2. What determines the accuracy level that is required for a survey?

   A. Time
   B. Weather
   C. Equipment
   D. Purpose

3. Measurement errors are classified as ________________.

   A. Systematic and accidental
   B. Continuous and retracted
   C. Horizontal and vertical
   D. Accidental and deliberate

4. What type of survey is used to gather data on natural and man-made land features?

   A. Construction
   B. Land
   C. Topographic
   D. Route

5. Elevation control is another term for ________________.

   A. Elevation
   B. Horizontal distance
   C. Angle of elevation
   D. Vertical control

LESSON 1

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct Answer and Feedback	
1.	B	Distance and direction A point may also be...(page 1-3, para 1-4)
2.	D	Purpose The purpose and type...(page 1-9, para 1-7c)
3.	A	Systematic and accidental Errors are classified as...(page 1-10, para 1-8)
4.	C	Topographic A topographic survey is conducted...(page 1-13, para 1-11b)
5.	D	Vertical control Vertical control (also called elevation control)...(page 1-15, para 1-12b)

LESSON 2

RATIOS, PROPORTIONS, ROOTS, AND POWERS

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn how to use mathematics.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn how to use ratios, proportions, roots, and powers.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Surveying operations require that you have a thorough understanding of mathematics to complete even the simplest surveying task. The fact that a calculator may accompany you on a survey mission does not excuse you from applying your knowledge of mathematics. As a surveyor, you need to know mathematics and must be able to make all the necessary computations in the following broad areas:

- Doing various kinds of field computations that are required to determine accurate lengths, geographic positions, and horizontal and vertical angles, as well as curves, grade lines, elevations, and earthwork volumes.

- Reviewing and checking all field data and values for completeness and accuracy.

In addition to the simple elements of mathematics, higher or more difficult forms of "math" are needed in surveying operations. In this lesson, you will refresh your understanding of ratios, proportions, roots, and powers. In the following lessons, you will study the more advanced branches of mathematics, such as geometry and trigonometry.

PART A - RATIOS AND PROPORTIONS

2-1. Defining Ratios and Proportions. You will find that a ratio can be expressed in four different ways. For example, the side-slope ratio of 2-to-1 is normally used with soft clay. This ratio can be expressed as follows: 2-to-1, 2:1, 2_1, or 2/1. The numbers 2 and 1, which are terms of the ratio, are called the antecedent and the consequent, respectively. The antecedent is the same as the dividend or numerator, and the consequent is the same as the divisor or denominator. Both terms of the ratio can be multiplied or divided by the same number without changing the value of the ratio. In the ratio 12/3, for example, the number 12 is divided by 3, giving the value of 4. This means that the ratio 12:3 is equal to the ratio 4:1. Other examples are shown below.

Example 1: What is the ratio of 6:2? Set up the problem as a fraction, and perform the indicated operation.

Solution: 6/2 = 3, or 3S1

Example 2: What is the ratio of 7:3? Set up the problem as a fraction, and perform the indicated operation.

Solution: 7/3 = 2 1/3, or 2 1/3:1

a. A proportion is a statement of equality between two ratios. If the value of one ratio is equal to the value of another ratio, they are said to be a proportion. For example, the ratio 3:6 is equal to the ratio 4:8. Therefore, this relationship can be written in one of the following forms:

(1) 3:6 :: 4:8

(2) 3:6 = 4:8

(3) 3/6 = 4/8

b. In any proportion, the first and last terms are known as the extremes and the second and the third terms are known as the means. If you look at the proportion example in the previous paragraph, you note that the terms "3' and "8" are the extremes, while the terms "6" and "4" are the means.

c. When working proportions, you should remember that there are three rules which are used in determining an unknown quantity. These rules can also be used to prove that the proportion is true.

(1) The first rule is that in any proportion the means' product equals the extremes' product. See the following examples:

(a) 2:3 :: 6:9

(b) 2 x 9 = 18 (extremes' product)

(c) 3 x 6 = 18 (means' product)

When the same problem is expressed in another form, the operation remains the same.

$$\frac{2}{3} = \frac{6}{9}$$

In this case, the proportion is expressed in a fractional form so that the numerator of one fraction is multiplied by the denominator of the other fraction. This is called the cross-multiplication process.

2 x 9 = 18 (extremes' product)
3 x 6 = 18 (means' product)

(2) The second rule is that in any proportion the means' product divided by either extreme gives the other extreme.

3:4 :: 6:8
4 x 6 = 24 (means' product)

$$\frac{24}{8} = 3 \text{ (other extreme)}$$

(3) The third rule is that in any proportion the extremes' product divided by either mean will determine the other mean.

2:3 :: 4:6
2 x 6 =12 (extremes' product)
12/3 = 4 (one mean)
12/4 = 3 (other mean)

d. The basic knowledge and appropriate use of the three proportion rules will aid you in determining the value of an unknown term when the other three terms are known. A typical use of proportion involves using the representative-fraction (RF) formula (RF = MD/GD), which is effective in determining ground distance (GD), map distance (MD), or the RF. Most, if not all, maps express their scales as a ratio or RF; that is, one unit on the map is equal to so many units on the ground. This map-to-ground relationship is extremely versatile since any kind of unit can be used to determine the map's RF. Thus, 1 inch on the map is equal to so many inches on the ground; the same ratio holds true for feet, meters, or yards. As a result, a map of a certain scale, or RF, may show that 1 foot on the map is equal to 20,000 feet on the ground. Therefore, the map's RF is 1:20,000. If a map of this scale were to be used, it would be easy to determine the actual distance (or linear measurement) of a survey line on the ground. See the following example:

Example: What is the length of a survey line on the ground if it measures 5 inches on a map having a RF of 1:20,000? Set up the problem using the RF formula (RF = MD/GD). Use the known information and write the proportion. Let x represent your unknown extreme.

RF = 1:20,000
MD = 5 inches
x = GD

Solution: 1:20,000 :: 5:x

Using the rule of the means' product divided by the known extreme (1), find the unknown extreme (x), which, in this case, represents the GD.

(5 x 20,000)/1 = 100,000

The answer is given in inches, which should be converted to the more conventional surveyor's units: yards, feet, meters, or miles. This requires that you divide the answer by 36, 12, 39.37, or 63,360, respectively.

e. The use of ratios and proportions can also serve you in other situations. See the following example:

Example: If a survey line measures 7,920 feet on the ground, what is the length of it on a map having a RF of 1:31,680? Set up your proportion using the RF formula (RF = MD/GD). Since the MD is usually expressed in inches, change the GD to inches by multiplying by 12. Let x equal the unknown mean.

RF = 1:31,680 (scale of progress map)
GD = 7,920 feet (length of surveyed line)
x = MD

Solution:

1:31,680 :: x:7,920 · 12
1:31,680 :: x:95,040

Using the rule of the extremes' product divided by the known mean (31,680), find the unknown mean (x), which, in this case, represents the desired MD.

(1 x 95,040) /31,680 = 3 inches

f. Up to this point, only direct proportion has been discussed. In a direct proportion, both ratios are direct ratios; that is, they increase or decrease in the same manner. In the map proportion problem above, the first ratio (1:31,680) increased in the same manner as the second ratio (3:95,040). This problem would also be a direct proportion if the terms had decreased in a like manner. Most of the proportion problems you will use are direct proportions and, unless specifically noted, will be treated as such. A simple clue that will aid you is to analyze each problem carefully to determine whether the unknown term will be greater or lesser than the known term of the ratio in which it occurs. Thus, each direct proportion takes the following pattern:

$$\frac{\text{lesser}}{\text{greater}} = \frac{\text{lesser}}{\text{greater}} \text{ or greater : lesser :: greater : lesser}$$

(increasing pattern) (decreasing pattern)

Therefore, when setting up a direct proportion problem, make sure that the ratio is stated correctly. Failure to do so may result in another type of proportion known as an inverse proportion, which will give you the wrong answer.

2-2. Defining Inverse Proportions. The term "inverse proportion" is given to those problems where an increase in the value of one term will cause a decrease in the value of another term. The same would hold true if a decrease in one term would cause an increase in another term. For example, the ratio 3:2 is the inverse of the ratio 2:3, thus when the two ratios are equated, the terms, or elements, are said to be inversely proportional. In this example, the product of each ratio is equal.

3:2 :: 2:3 or 2:3 :: 3:2

a. Another example is where the means of a proportion can be changed in an inverse manner while the extremes are held constant (the same can be done with the extremes and the means held constant). For example, in the proportion 2:5 :: 8:20, the means' product (5 x 8) is 40. If the first mean is doubled and the second mean is halved, the proportion becomes 2:10 :: 4:20. The means' product is still 40 even through the means were inversely changed. You will note that the means' product of both proportions is equal to the extremes' product; thus, the proportional relationship between the ratios remains the same. A similar situation exists if the extremes are changed.

2:5 :: 8:20

Doubling the first extreme and halving the remaining extreme changes the proportion but not the relationship between ratios.

4:5 :: 8:10

Any of the three proportion rules may be applied to the examples above to determine an unknown value and to prove that a proportion is true.

b. Proportion problems can be used effectively in determining the manpower needed on a proposed survey project For example, it is necessary to estimate the number of days needed to survey a pipeline right-of-way in a tropical country. Although the line is long, this assignment normally takes 252 days for 2 survey teams to complete. However, adverse weather conditions are expected in about 60 days; therefore, can 12 survey teams complete the assignment before day 60?

c. Your analysis should indicate that if the number of survey teams increases, the number of days to complete the assignment will decrease. Therefore, in order to solve this problem, use an inverse proportion, whereby one term decreases as another term increases. The best way to set up an inverse proportion is to equate like terms, such as 2 teams:12 teams and 252 days:x days. You infer that the time involved with 12 survey teams is less than that required with 2 teams therefore, you can invert the time ratio within the proportion and carry out its solution.

2:12 :: x:252

d. By using the rule of the extremes' product divided by the known mean to determine the unknown mean, you will find that 12 survey teams can accomplish the task in 42 days--well before day 60.

PART B - ROOTS AND POWERS

2-3. Defining Roots and Powers. If you were asked to multiply 842 by itself and 369 by itself, it would mean that both of these numbers were raised to their second powers. Actually, a number can be raised to any power, and this fact is indicated by an exponent. The exponent is a number placed somewhat above and to the right of the number. Thus, 842 x 842 can be written in its proper form of 842^2. Other powers of a number are indicated by the appropriate exponent. For example, 5 x 5 x 5 x 5 x 5, which is written 5^5, means that the number 5 must be raised to its fifth power. Thus, the power indicates a special case of multiplication and enables you to express numbers in a form that is useful for special application.

a. If the power is a low-value integer, it can be readily computed by arithmetic methods. Powers that are large numbers or decimals are usually determined through the use of logarithm tables or, if available, computers. Low-value exponents are frequently in many of the field-survey computations, particularly where known values are substituted for lettered terms that often must be squared (raised to the second power). One of the formulas that a surveyor frequently uses while in the field is $a^2 + b^2 = c^2$; each letter represents a side of a right triangle (one that contains a 90° angle). Generally, a surveyor can lay out a right triangle and measure two sides where some obstacle or obstruction prevents measuring the third side. Thus, if sides a and b are known, it is simple to find the value of c.

b. Equal factors of a number are known as roots; therefore, finding the root of a number is the reverse of finding the power. When two equal factors are found for a number, each is known as the second root, or square root. For example, the square root of 25 is 5, since 5 x 5 or $5^2 = 25$. When three equal factors are found, each factor is known as a third root, or cube root. The cube root of 64 is 4, since 4 x 4 x 4 or $4^3 = 64$. Four equal factors are known as a fourth root, five equal factors are known as a fifth root, and so on.. Square and cube roots can readily be determined by an arithmetic method. However, only square roots are normally found by this method, since cube-root determinations are very involved and time consuming.

c. In determining the root of a number, you must use these two notations; the radical sign and the exponent. The radical sign ($\sqrt{}$) is combined with the vinculum (___) and placed in front of a number to indicate that an extraction of its square root is necessary. When a root other than a square root needs to be determined, this is indicated by a small index number placed in the angle formed by the radical sign as shown below.

$$\sqrt[3]{216} = 6$$

$$\sqrt[4]{81} = 3$$

$$\sqrt[5]{16807} = 7$$

d. Often, you will find that the exponent is used to show the particular root extraction of a number that is desired. For example, the square-root extraction of a number can be indicated by the exponent 12, the cube root by 1/3, the fourth root by 1/4, the fifth root by 15, and so forth. Examples of the equal factor or root determination by exponents is shown below. In each example, the root 3 was needed.

9^{2} (9 = 3 x 3)
$27^{1/3}$ (27 = 3 x 3 x 3)
$81^{1/4}$ (81 =3 x 3 x 3 x 3)
$243^{1/5}$ (243 = 3 x 3 x3 x 3 x 3)

e. According to your proficiency of common multiplication, you can determine mentally the square root of some numbers. For example, the square root of $25^{1/2}$ is 5, since 5 x 5 or $5^2 = 25$. Similarly, the square root of $144^{1/2}$ is 12, since 12 x 12 or $12^2 = 144$. In other cases, however, the square root of a number must be determined by a mathematical process. If the number is a perfect square, the square root will be an integral number; if the number is not a perfect square, the square root will be a continued decimal.

f. Currently, there are tables that are designed to determine the root of any number. However, if these tables are unavailable to you while in the field, you must be able to determine the square root of any number. For example, suppose that you have just laid out one leg of a triangle and want to calculate the square root. The length of the leg (or side) is 3,398.89 feet. The first step would be to separate the number (3,398.89) into 2-digit groups, starting from the decimal point and working in both directions.

$$\sqrt{3398.89}$$

g. Next, you will place the decimal point for the intended answer directly above the decimal point that appears in the dividend under the radical sign. The square root for this number will have 1 digit for each 2-digit group. Now determine the largest number that can be squared without exceeding the first pair of digits (33). The answer is 5, since the square of any number larger than 5 will be greater than 33. Place the digit 5 above the first pair of digits in the dividend. Squaring 5, place the product under the first two digits (33), and perform the indicated subtraction. Now bring down the next pair of digits as shown below.

```
  5
√3398.89
 25
  898      (new dividend)
```

h. To obtain the next trial divisor, the digit 5 is doubled and placed to the left of the new dividend. Divide the trial divisor (10) into all but the last digit of the new dividend; it will go into 89 eight times. Place the number 8 above the second pair of digits, and also place the number 8 to the right of the trial divisor. Thus, the new trial divisor is now 108. Multiply the new trial divisor by 8, and write the product just under the former remainder (898). Perform the indicated subtraction, and bring down the next pair of digits (89).

	5 8.	
	$\sqrt{3398.89}$	
	25	
5 x 2 = 10(8)	898	
8 x 108 =	864	
	34 89	(new dividend)

i. To obtain the next trial divisor, double the partial answer (58). Divide the trial divisor (116) into all but the last digit of the new dividend; it will go into 348 three times. Place the digit 3 to the right of the decimal point in the quotient immediately above the third pair of digits in the original dividend. Also, place the 3 to the right of the trial divisor. Thus, the true divisor is 1163. Multiply 1163 by 3 to obtain a product of 3,489, which is written under the new dividend. Since there is no remainder, you may consider the figure 3,398.89 as a perfect square, and its square root is 58.3.

	5 8. 3	
	$\sqrt{3398.89}$	
	25	
	898	
	864	
2 x 58 = 116(3)	34 89	
3 x 1163 =	34 89	(new dividend)

j. The final step in the square-root-extraction process should be a check of your work. This check can be done by squaring 58.3 and arriving at the dividend shown under the radical sign.

LESSON 2

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. There is only one correct answer to each item. When you complete the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part of the lesson which contains the portion involved.

1. 2^7 = _______.

    A. 7x7
    B. 2x2x2x2x2x2
    C. 14
    D. 128

2. The number 6 in the expression 3^6 is called the ________.

    A. Square
    B. Exponent
    C. Cube
    D. Proportion

3. What is the ratio of 5-to-3?

    A. 15-to-6
    B. 3-to-1
    C. .06-to-1
    D. 1.67-to-1

4. The expression 6-to-9 = 5-to-8 is referred to as being ________.

    A. A process
    B. A ratio
    C. A proportion
    D. invalid

5. What is the value of the missing extreme in the expression 2:12,000 =3:________?

    A. 18,000
    B. 6,000
    C. 24,000
    D. Cannot be determined

LESSON 2

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct answer and feedback	
1.	D	128 Equal factors of a number are known...(page 2-6, para 2-3b)
2.	B	Exponent The exponent is a number...(page 2-6, para 2-3)
3.	D	1.67-to-1 You will find that a ratio can be expressed...(page 2-2, para 2-1)
4.	C	Proportion If the value of one ratio is equal...(page 2-2, para 2-1a)
5.	A	18,000 The first rule is that in any proportion...(page 2-2, para 2-1c(2)

LESSON 3

GEOMETRY

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn how to use geometry.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn how to use geometry for surveying operations.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Geometry deals with lines, angles, surfaces, and solids. In general, the lengths of lines or the sizes of angles (measurable in degrees or parts of a degree) are not measured by divisions on a ruler but are compared with one another. One of the most important uses of geometry is understanding other branches of math, which are needed to solve problems relating to geodetic positions, elevations, areas, or volumes of earthwork or other material. Also, understanding geometry helps in understanding trigonometric principles. A surveyor will use geometry in his daily work.

PART A - LINES

3-1. General Information. A line is generated by a point in motion; it has the dimension of length but not thickness. Dots and lines made on a drawing actually have thickness and are merely convenient representatives of points and lines. A line that has the same direction for its entire length is called a straight line. A line that changes in direction along its length is called a curved line. A flat surface is generated by a straight line moving in a direction other than its length. A surface has the dimensions of length and width but not thickness.

PART B - ANGLES

3-2. General Information. An angle can be identified by the symbol for its vertex. In Figure 3-1, the vertex is called angle O. This method can be used if there are no other angles at point O to cause confusion. However, in Figure 3-2, four angles have their vertex at point O, therefore, this method cannot be used. The letters of the sides and vertex are commonly used in geometry to identify angles. This method is more exact and should leave no question as to which angle is referenced. In Figure 3-1, the angle can be identified as AOB. The symbol $\angle$ can be used in place of the word "angle." Thus, angle AOB can be written as $\angle$AOB or $\angle$O. A symbol, such as the Greek letter θ, can be used to identify an angle. This method is generally used in trigonometry.

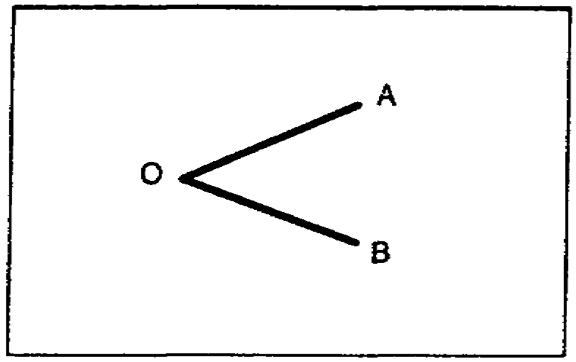


Figure 3-1. Angle O

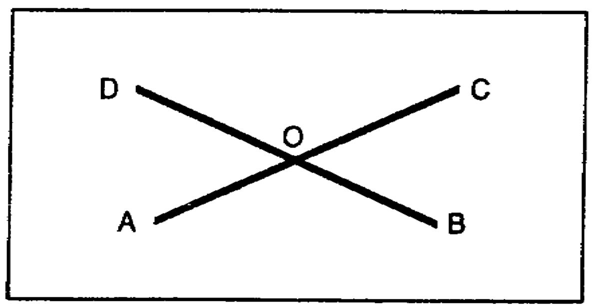


Figure 3-2. Angle AOB

Figure 3-3 shows the five general classes of angles. An angle that-

- Is formed by perpendicular lines is a right angle.
- Is less than a right angle is an acute angle.
- Is greater than a right angle but less than a straight angle is called an obtuse angle.
- Has sides which extend in opposite directions from the vertex is called a straight angle.
- Is greater than a straight angle but less than two straight angles is called a reflex angle.

Angles can also be classified into two general groups: right and oblique. Oblique angles are all angles other than straight and right angles.

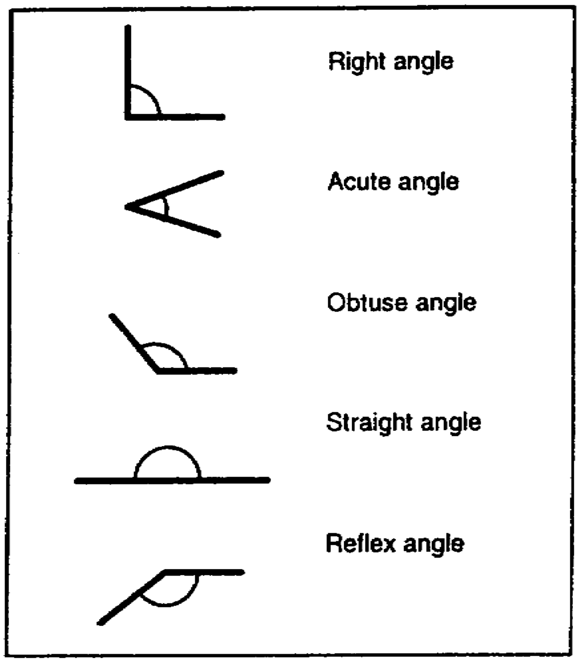


Figure 3-3. Classes of angles

a. Adjacent Angles. Angles that have a common vertex and a common side between them are called adjacent angles. Thus, in Figure 3-2, BOC and COD are adjacent angles; however, BOC and AOD are not adjacent angles because they have no common side. When one straight line meets another straight line to form two equal adjacent angles, the lines are perpendicular to each other, and the angles are right angles. In Figure 3-4, line CO is perpendicular to line AB, and angles BOC and AOC are right angles. The small square at the point where the lines meet is used to indicate that a right angle is formed by the two lines.

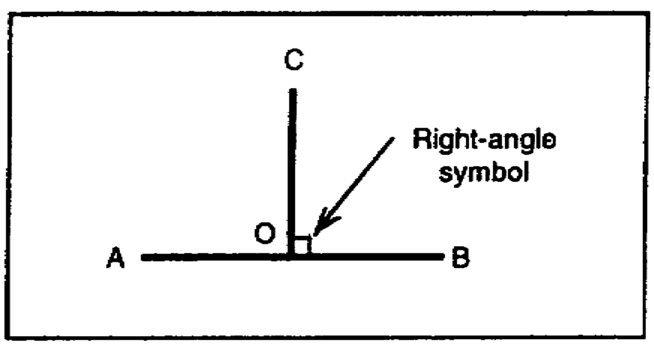


Figure 3-4. Right angles

b. Related Angles. General relationships between angles are shown in Figure 3-5, page 3-4. When two combined angles are equal to a right angle, they are called complementary angles. In Figure 3-5, angle AOC is a right angle, and angles AOB and BOC are complementary angles. Angle AOB is the complement of BOC, and angle BOC is the complement of AOB. When two combined angles are equal to a straight angle, they are called supplementary angles. In Figure 3-6, angle AOB is a straight angle, and angles AOC and BOC are supplementary angles. Angle AOC is the supplement of BOC, and angle BOC is the supplement of AOC. When two lines intersect, the opposite angles are called vertical angles. In Figure 3-7, angles AOD and BOC are vertical angles, and angles AOC and BOD are also vertical angles.

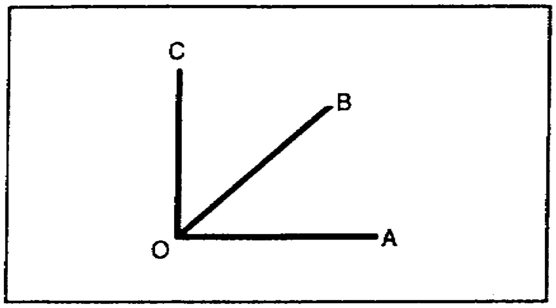


Figure 3-5. Complimentary angles

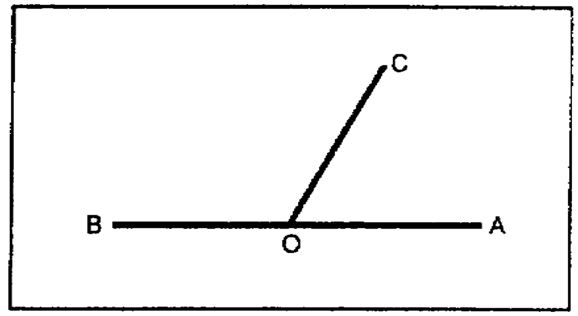


Figure 3-6. Supplementary angles

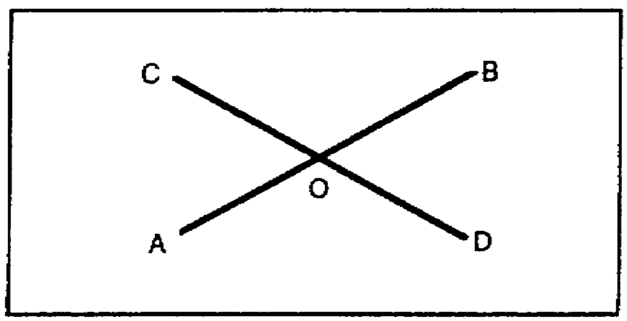


Figure 3-7. Vertical angles

3-3. Measuring Angles. Measuring angles is important in calculations for surveying problems. Degrees (°) are used for angle measurements. An angular degree is 1/360th of a circle. The degree is further divided into minutes (') and seconds ("), with 60 minutes in 1° and 60 seconds in 1 minute.

3-4. Generating Angles. You can generate an angle by rotating a line as shown in Figure 3-8. Rotate line OP in a counterclockwise direction about point O, starting from line AO. By this action the angle AOP is generated. When line OP is at one-quarter of a revolution, it is perpendicular to line AO, and a right angle is formed. When you rotate line OP completely around the circle, it generates four right angles (360°). Thus, the total angular magnitude about a point in a place is equal to four right angles.

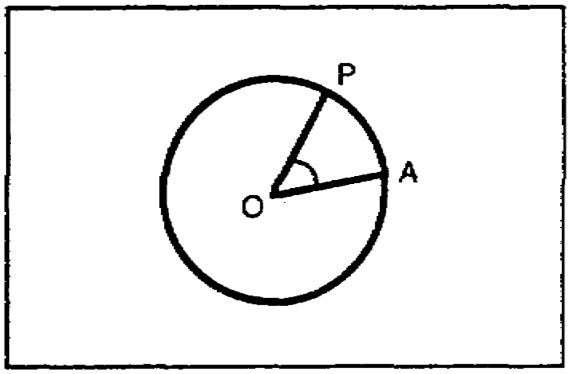


Figure 3-8. Generating an angle

PART C - TRIANGLES

3-5. General Information. A portion of a plane that is bounded by three straight lines is a plane triangle, or simply a triangle. The lines that bound the triangle are called its sides, and the sum of their lengths is its perimeter.

a. An angle that is formed within a triangle by any two sides is called an interior angle. An angle that is formed outside a triangle by any side and the extension of another side is called an exterior angle. Interior and exterior angles of a triangle are shown in Figure 3-9. Whenever the angles of a triangle are mentioned, the interior angles are referenced unless otherwise specified. The interior angles that are not adjacent to a given exterior angle are called opposite interior angles.

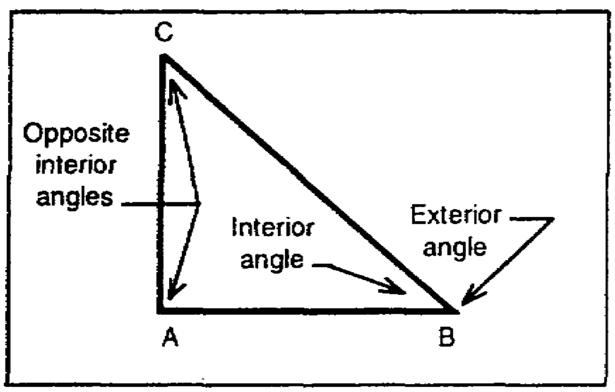


Figure 3-9. Angles

b. One side of a triangle is usually drawn as a horizontal line with the other two sides above it. The bottom side is then called the base of the triangle, as shown in Figure 3-10, page 3-6. However, any side of a triangle can be designated as its base, and it is not necessary to draw it at the bottom of the figure. The angle opposite the base is called the vertex angle, and the vertex of this angle is called the vertex of the triangle. The perpendicular distance from the vertex of a triangle to its base, or to an extension of its base, is called the altitude of the triangle. The side of a triangle that is opposite a right angle is called the hypotenuse.

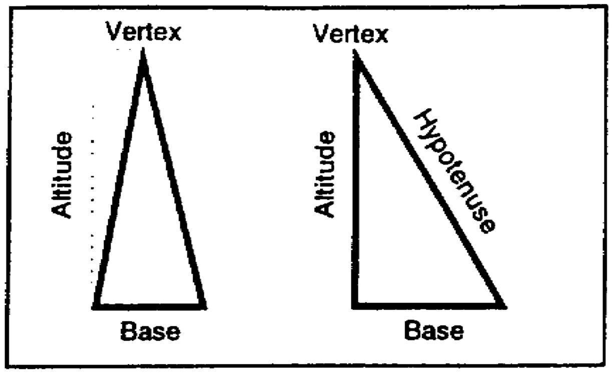


Figure 3-10. Sides and altitude

c. The general types of triangles are shown in Figure 3-11. They are as follows:

(1) An acute triangle has interior angles that are less than right angles.

(2) A right triangle has one right angle and two acute angles.

(3) An obtuse triangle has one obtuse angle and two acute angles.

(4) An isosceles triangle has two equal sides and two equal angles.

(5) An equilateral triangle has three equal sides and three equal angles and is sometimes called an equiangular triangle. All of the angles within an equilateral triangle are less than right angles; therefore the equilateral triangle is also an acute triangle.

d. Obtuse and acute triangles are often called oblique triangles to distinguish them from right triangles. Triangles with three unequal sides are called scalene triangles.

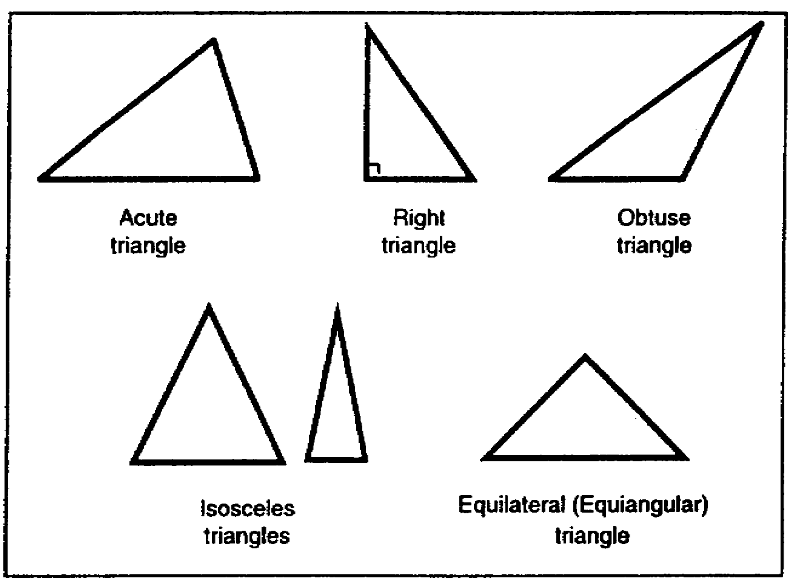


Figure 3-11. Types of triangles

PART D - POSTULATES

3-6. Basic Concepts of Postulates. Fundamental principles of geometry that depend on personal observation and must be taken without proof are called postulates. Postulates are used to develop rules for other branches of mathematics and to prove many arithmetic and algebraic relationships. The basic postulates are that-

a. A straight line is the shortest distance between two points.

b. Only one straight line can be drawn between the same two points.

c. Two straight lines cannot intersect at more than one point.

d. A straight line may be extended indefinitely.

e. A straight line may be drawn from a point to any other point.

f. A geometric figure may be moved from one position to another without any change in form or magnitude.

g. All right angles (90° angles) are equal.

h. All straight angles (180° angles) are equal.

i. Only one perpendicular line can be drawn from a point in a line in a plane.

j. Only one perpendicular line can be drawn from a point outside a line in a plane.

k. Two adjacent angles which have their exterior sides in a straight line are called supplementary angles.

l. The sum of all angles about a point on one side of a straight line in a plane is equal to two right angles (180°).

m. The sum of all angles about a point in a plane is equal to four right angles (360°).

n. Angles that have the same supplement are equal, and angles that have the same complement are equal.

o. Vertical angles are equal.

p. A line segment can be bisected in only one point.

q. An angle can be bisected by only one line.

r. The sides of a square are equal.

s. Only one circle can be drawn with a given point as center and a given distance as radius in a plane.

t. All radii of the same circle or of equal circles are equal.

u. All diameters of the same circle or of equal circles are equal.

v. A straight line can intersect a circle at only two points. If the two points coincide, the straight line is tangent to the circle.

w. A circle can intersect another circle at only two points.

x. A diameter bisects a circle.

y. Only one line parallel to a given line passes through a given point.

3-7. Superposition. Two geometric magnitudes that coincide exactly, when one is placed upon the other, are equal. This method of establishing equality is called the method of superposition. In practice you will not actually move the geometric magnitudes but merely compare them mentally. If you conceive that one straight line can be placed upon another straight line so that the ends of both coincide, the lines are equal. If you determine that one angle can be placed over another angle so that their vertices coincide and their sides go in the same directions, the angles are equal. If you determine that one figure can be placed upon another figure so that they coincide at all places, the figures are equal. Figures that coincide exactly when superposed are congruent.

PART E - THEOREMS FOR LINES, ANGLES, AND TRIANGLES

3-8. Basic Concepts of Theorems and Corollaries. A geometric rule that can be proved by using postulates, illustrations, and logical reasoning is called a theorem. A secondary rule whose truth can be easily deducted from a theorem is called a corollary. You can use theorems and corollaries to solve geometric problems without having to use the basic relationships that are established by the postulates.

3-9. Theorems for Lines. The following theorems show the relationship between straight lines in the same plane.

a. Theorem 1. Only one perpendicular line can be drawn from a given line to a given point outside that line. A corollary for this theorem is that a perpendicular line is the shortest line that can be drawn from a given line to a given point.

b. Theorem 2. Two lines in the same plane and perpendicular to the same line are parallel. Therefore, if a straight line is perpendicular to one of two parallel lines, it is also perpendicular to the other. This relationship is shown in Figure 3-12.

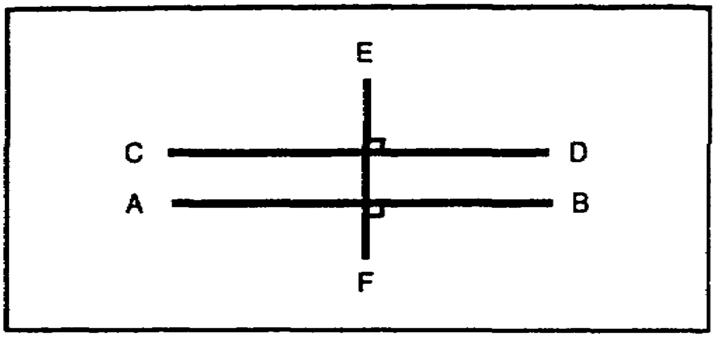


Figure 3-12. Parallel lines cut by a perpendicular line

c. Theorem 3. If two parallel lines are cut by a transversal, the alternate interior angles are equal. Therefore, when two lines are cut by a transversal and one pair of alternate interior angles are equal, the two lines are parallel. In Figure 3-13, lines AB and CD are parallel and line EF is the transversal. Interior angles r and r^1 are equal, and interior angles s and s^1 are equal.

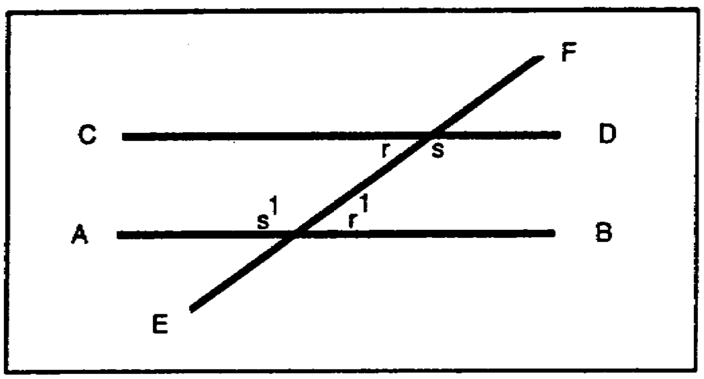


Figure 3-13. Parallel lines cut by a transversal

d. Theorem 4. Straight lines that are parallel to the same line are parallel to each other. In Figure 3-14, if line CD is parallel to AB and line EF is parallel to AB, then line CD is parallel to EF.

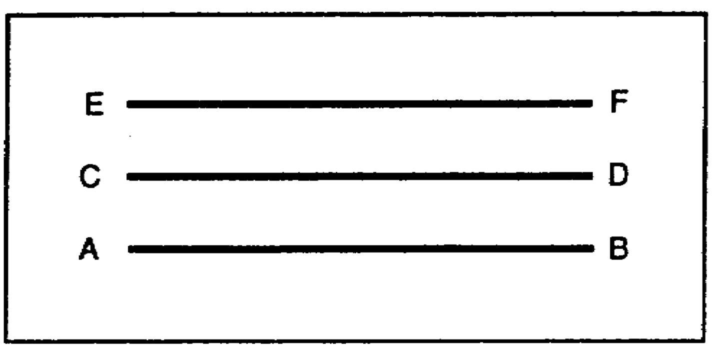


Figure 3-14. Parallel lines

e. Theorem 5. Any point on a perpendicular bisector of a line segment is equidistant from the extremities of the line segment. The distances from any point not on the perpendicular bisector to the extremities of the line segment are unequal. Figure 3-15 shows that line AC is equal to BC and line AD is equal to BD, but line AE is not equal to BE.

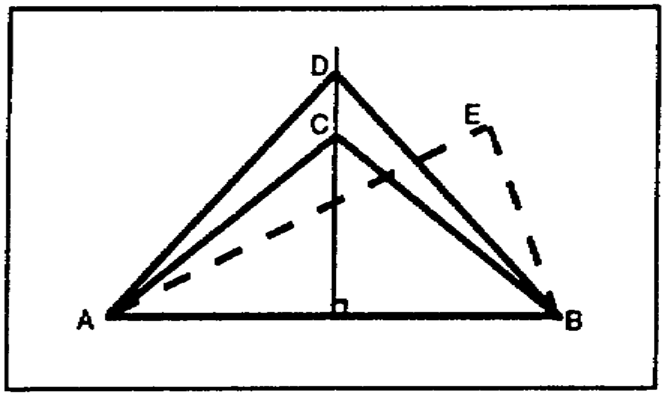


Figure 3-15. Points equidistant from line segment extremities

3-10. Theorems for Angles. The following theorems show the relationship of two angles whose respective sides are either parallel or perpendicular to each other.

a. Theorem 1. If two angles have their sides respectively parallel, the angles are either equal or supplementary, as shown in Figure 3-16. Line C^1C^{11} is parallel with AC, and line A^1B^1 is parallel with AB. Angle $B^1A^1C^1$ is equal to BAC, and angle $B^1A^1C^{11}$ is the supplement of $B^1A^1C^1$; therefore, angle $B^1A^1C^{11}$ is also the supplement of BAC.

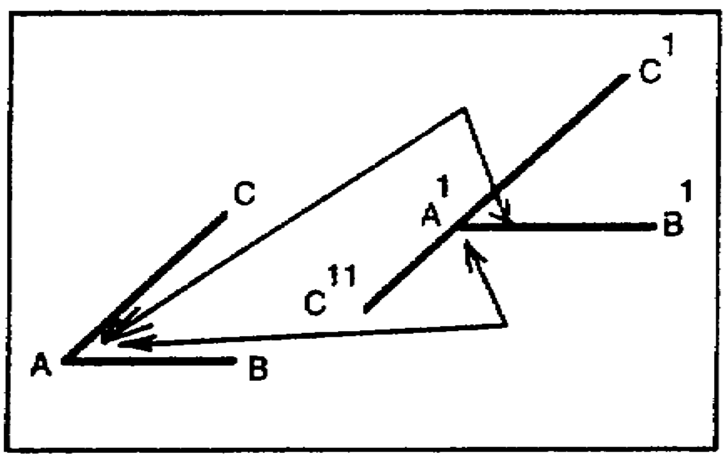


Figure 3-16. Parallel sides

b. Theorem 2. If two angles have their sides respectively perpendicular, the angles are either equal or supplementary, as shown in Figure 3-17. Line C^1C^{11} is perpendicular to AC, and line A^1B^1 is perpendicular to AB. Angle $B^1A^1C^1$ is equal to BAC, and angle $B^1A^1C^{11}$ is the supplement of $B^1A^1C^1$; therefore, angle $B^1A^1C^{11}$ is also the supplement of BAC.

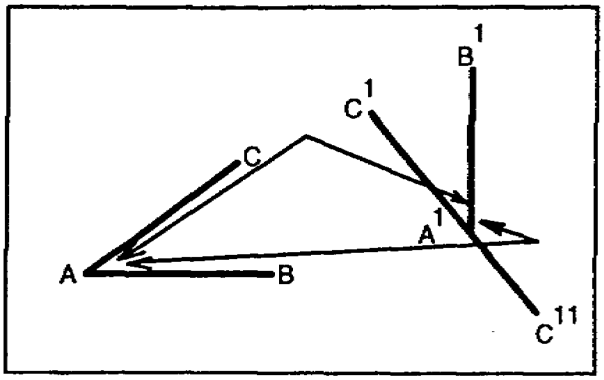


Figure 3-17. Perpendicular sides

c. Theorem 3. Every point on an angle bisector is equidistant from the sides of the angle. The distances from a point not on the angle bisector to the sides of the angle are unequal. Figure 3-18 shows that line AB is equal to BC and line DE is equal to EF, but line GP is not equal to PH. A corollary for this theorem is that all points within an angle that are equidistant from its sides lie in the angle bisector.

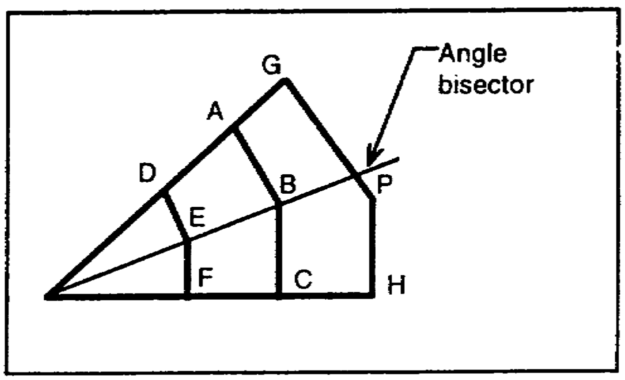


Figure 3-18. Points equidistant from the sides of an angle

3-11. Theorems for Triangles. The following theorems are for the basic relationships of the angles and sides of a triangle.

a. Theorem 1. The sum of the angles of a triangle is equal to two right angles. This theorem can be proved by using the illustration shown in Figure 3-19, page 3-12. Line AB is extended to D, and line BE is parallel to AC. Angle r is equal to w, and angle t is equal to u. Since angle s plus angle u plus angle w equals 180°, angle r plus angle t plus angle s also equals 180° or two right angles. The following corollaries can be developed from this theorem:

(1) The sum of the two acute angles of a right triangle is equal to a right angle.

(2) A triangle cannot have more than one right angle or more than one obtuse angle.

(3) The third angles of two triangles are equal if the two angles of one triangle are equal to the two angles of the other triangle.

(4) The right triangles are equal if the side and the acute angle of one triangle are equal to the corresponding side and the acute angle of the other triangle.

(5) Any exterior angle of a triangle is equal to the sum of the two opposite interior angles.

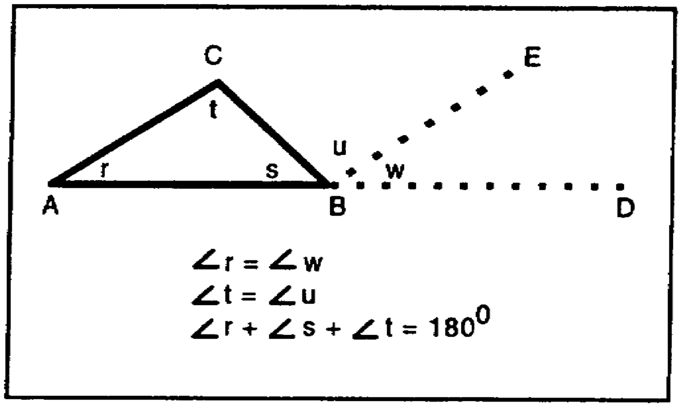


Figure 3-19. Sum of angles equal 180°

b. Theorem 2. Any side of a triangle is less than the sum of the other sides. This theorem can be proved by using the postulate in paragraph 3-6a. As shown in Figure 3-20, side AB is a straight line and the other two sides, AC and CB, form a bent line between points A and B.

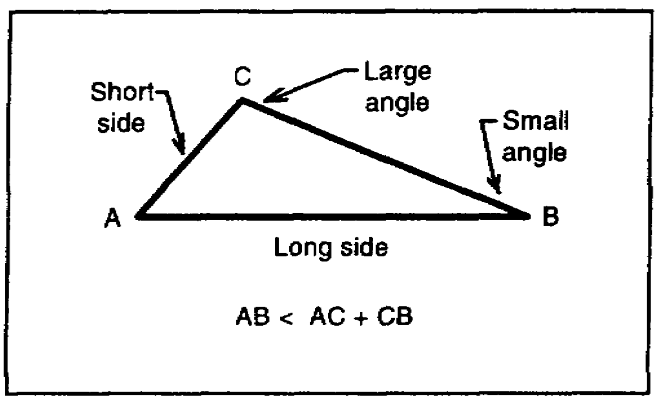


Figure 3-20. Relative sizes of angles and sides

c. Theorem 3. If two sides of a triangle are equal, the angles opposite these sides are equal, as shown in Figure 3-21. Conversely, if two angles of a triangle are equal, the sides opposite these angles are equal. The following corollaries can be derived from this theorem:

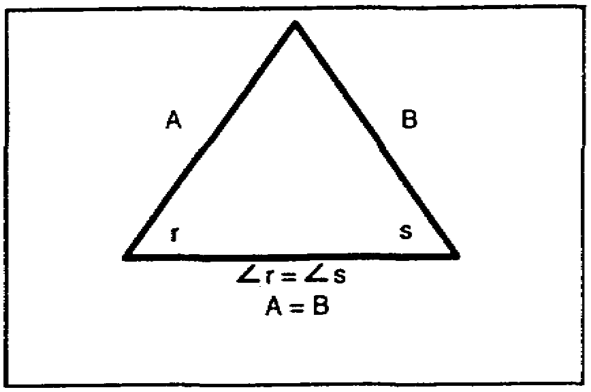


Figure 3-21. Two equal sides

(1) An equilateral triangle is also an equiangular triangle.

(2) The bisector of the angle opposite the unequal side of an isosceles triangle is the perpendicular bisector of the base line.

(3) The perpendicular bisector of the unequal side of an isosceles triangle bisects the opposite angle.

d. Theorem 3. If one side of a triangle is longer than another side (short side), the angle opposite the long side is greater than the angle opposite the short side. This relationship is shown in Figure 3-20.

3-12. Theorems for Triangle Bisectors, Altitudes, and Medians. The following theorems are for the relationships of the triangle bisectors, altitudes, and medians.

a. Theorem 1. If a line is parallel to one side of a triangle and bisects the other two sides, it is half as long as the side to which it is parallel. In Figure 3-22, line DC is parallel to AB, line EC is equal to BC, line ED is equal to AD, and line DC is one-half the length of AB.

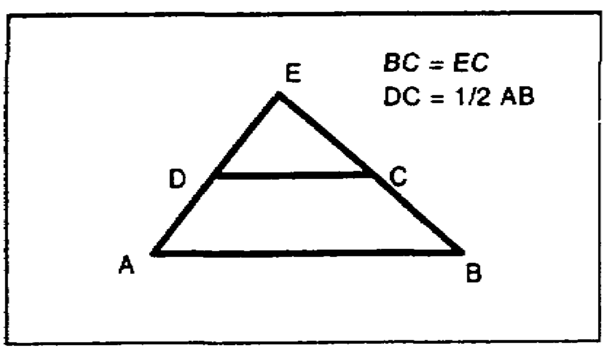


Figure 3-22. Line parallel to one side and bisecting the other two sides

b. Theorem 2. Perpendicular bisectors of the sides of a triangle meet in a point that is equidistant from the three vertices of the triangle. In Figure 3-23, page 3-14, point P is where the perpendicular bisectors meet.

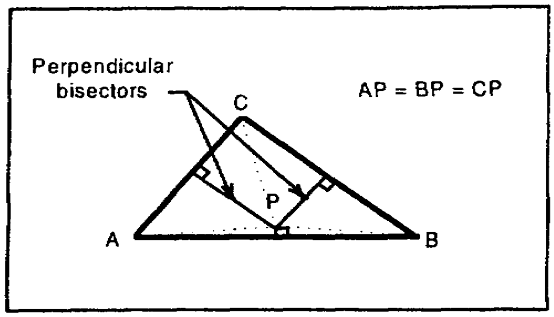


Figure 3-23. Perpendicular bisectors

c. Theorem 3. The altitudes of a triangle meet in a point. In Figure 3-24, point P is where the altitudes meet.

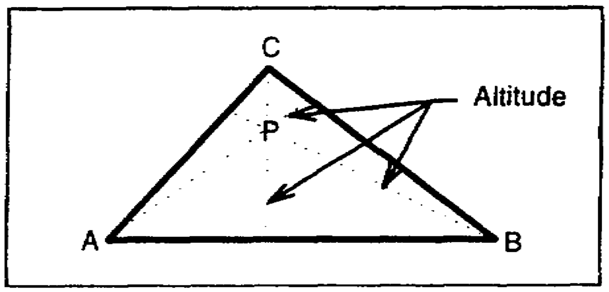


Figure 3-24. Altitudes

d. Theorem 4. The angle bisectors of a triangle meet in a point that is equidistant from the three sides. Figure 3-25 shows that the angle bisectors of a triangle meet at point P. Perpendicular lines from the sides (XP, YP, and ZP) are of equal length.

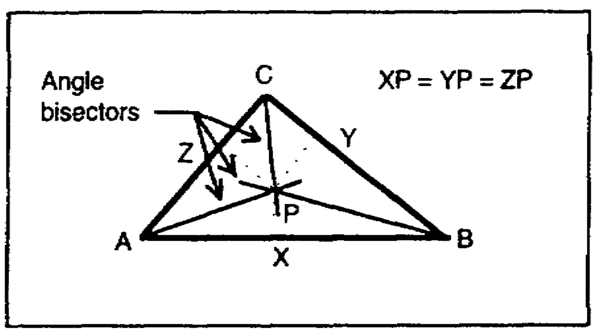


Figure 3-25. Angle bisectors

e. Theorem 5. The medians of a triangle meet at a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side, see Figure 3-26. A median is a line from the vertex to the midpoint of the opposite side of a triangle. Point P is called the centroid of the triangle because this point is the center of gravity.

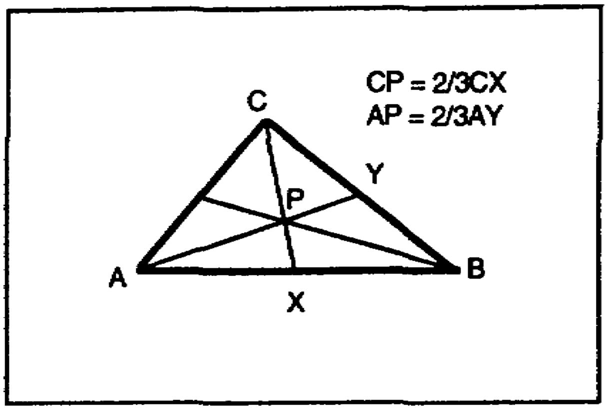


Figure 3-26. Medians

3-13. Theorems for Congruent Triangles. The following theorems can be used to determine whether two triangles coincide exactly when superposed.

a. Theorem 1. Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, as shown in Figure 3-27.

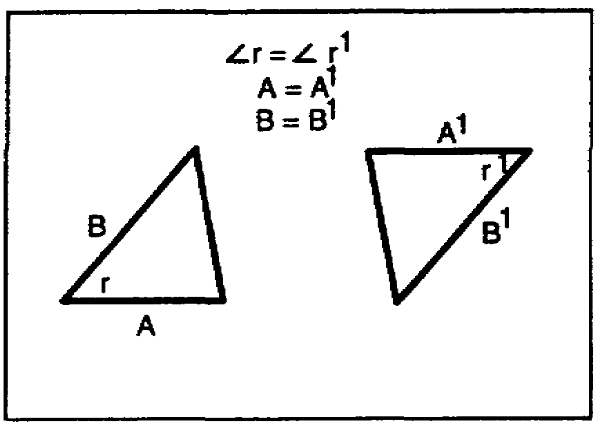


Figure 3-27. Triangles with two sides and the included angle equal

b. Theorem 2. Two triangles are congruent if a side and the adjacent angles of one triangle are equal to a side and the adjacent angles of the other triangle, as shown in Figure 3-28, page 3-16.

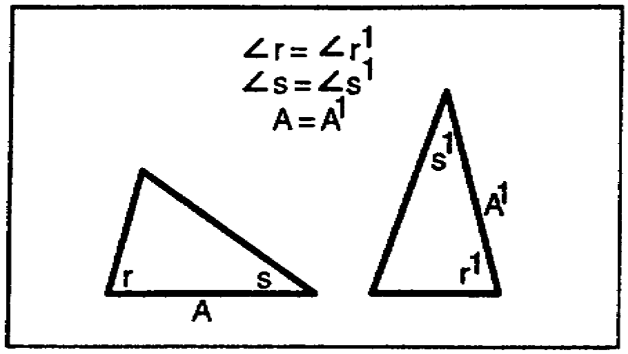


Figure 3-28. Triangles with a side and adjacent angles equal

c. Theorem 3. Two triangles are congruent if the three sides of one triangle are equal, respectively, to the three sides of the other triangle, as shown in Figure 3-29.

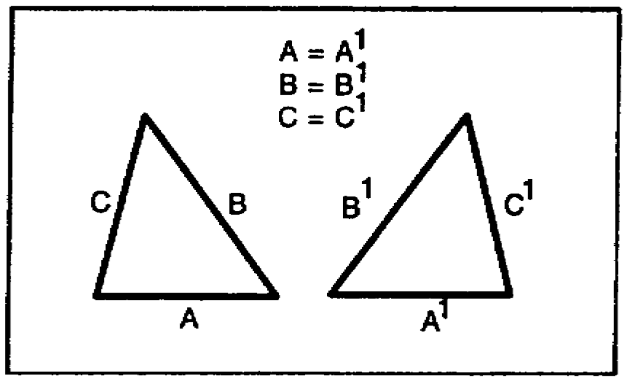


Figure 3-29. Triangles with three sides equal

d. Theorem 4. Two right triangles are congruent if the hypotenuse and a leg of one triangle are equal to the hypotenuse and a leg of the other triangle, as shown in Figure 3-30.

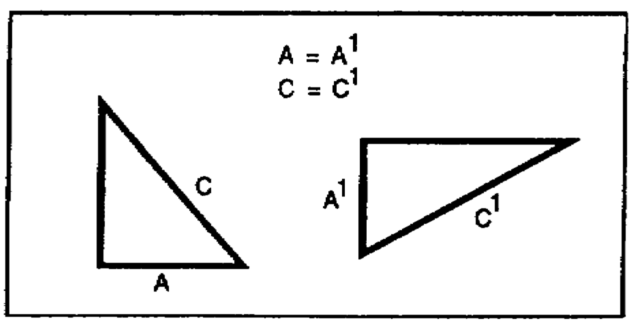


Figure 3-30. Right triangles with two ides equal

3-14. Theorem for the Sides of a Right Triangle. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. This relationship is known as the Pythagorean theorem.

a. The Pythagorean theorem can be proved by using the illustration shown in Figure 3-31. Examples are as follows:

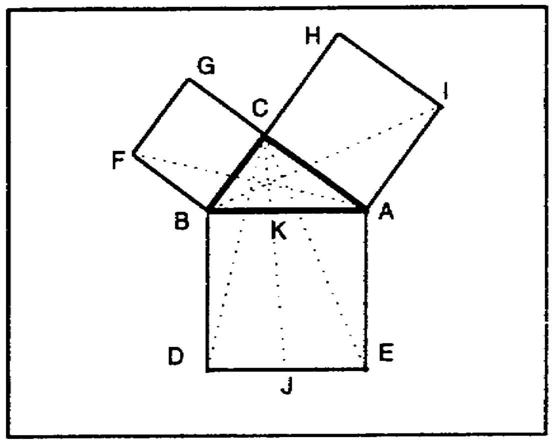


Figure 3-31. Any right triangle

(1) Line AE is equal to AB and line Al is equal to AC of triangles CAE and IAB because these lines are the sides of the squares.

(2) Angles EAC and BAI are equal.

(3) Triangles EAC and BAI are equal.

(4) AC is the altitude and AI is the base of both square AIHC and triangle AIB.

(5) The area of triangle AIB is one-half the area of square AIHC.

(6) AD is the altitude and AE is the base of both rectangle AKJE and triangle ACE.

(7) The area of triangle ACE is one-half the area of rectangle AKJE. Therefore, the areas of rectangle AKJE and square AIHC are equal.

(8) Line BD is equal to AB and line BC is equal to BF of triangles CBD and FBA because these lines are the sides of the squares.

(9) Angles CBD and FBA are equal.

(10) Triangles CBD and FBA are equal.

(11) BC is the altitude and FB is the base of both square FBCG and triangle FBA.

(12) The area of triangle FBA is one-half the area of square FBCG.

(13) BK is the altitude and BD is the base of both rectangle BDJK and triangle CBD.

(14) The area of triangle CBD is one-half the area of rectangle BDJK. Therefore, the areas of rectangle BDJK and square FBCG are equal.

(15) The area of square ABDE is equal to the sum of the areas of the two rectangles (AKJE and BDJK). Therefore, the area of square ABDE is also equal to the sum of the two squares (AIHC and FBCG).

b. A special case of the right triangle, where each side can be divided into an integral number of basic units, is shown in Figure 3-32. The length of side A is three units, the length of side B is four units, and the length of the hypotenuse (side C) is five units. This triangle, called a 3-4-5 right triangle, is often used to mark out long sides with an included right angle. Triangles with sides that are multiples of 3, 4, and 5 (such as 15, 20, and 25) are also considered to be 3-4-5 right triangles. There are other right triangles whose sides can be divided into an integral number of basic units, one of which has sides that are 5, 12, and 13 units long. However, the 3-4-5 triangle is the easiest to remember and the most commonly used.

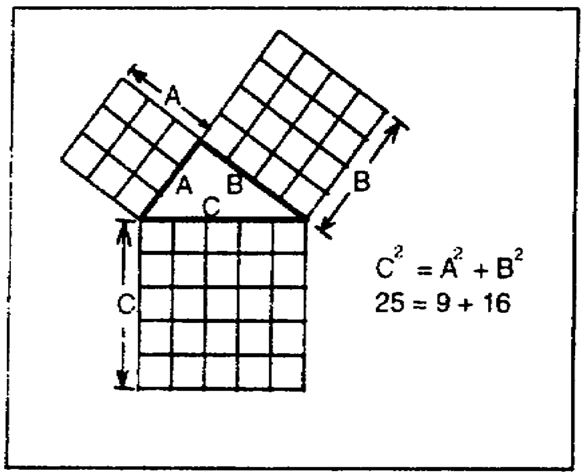


Figure 3-32. Right triangle

3-15. Theorems for Similar Triangles. The corresponding angles of similar triangles are equal. Equal triangles are similar, but similar triangles are not necessarily equal. The important relationship of similar triangles is that a direct proportionality exists between corresponding sides. The following theorems can be used to determine whether triangles are similar.

a. Theorem 1. A line parallel to one side of a triangle and intersecting the other two sides divides these two sides proportionally. This condition is shown in Figure 3-33, in which the following proportion exists:

AD : DC = BE : EC

Conversely, if a line divides two sides of a triangle proportionally, it is parallel to the third side.

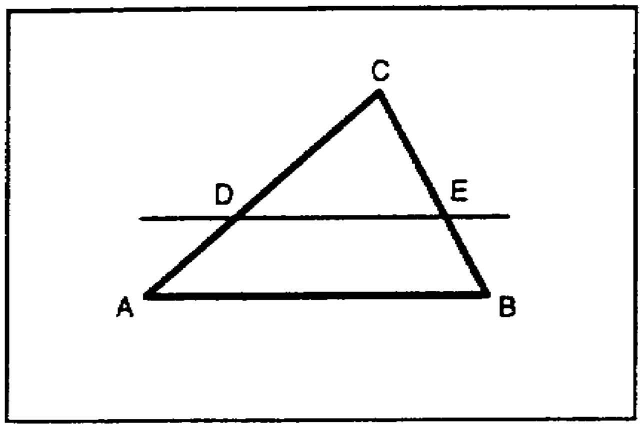


Figure 3-33. Line parallel to one side

b. Theorem 2. If the three angles of one triangle are equal to the three angles of another triangle, the triangles are similar. Similar triangles are shown in Figure 3-34. Since the sum of the interior angles of a triangle is 180°, the following corollaries can be developed from this theorem:

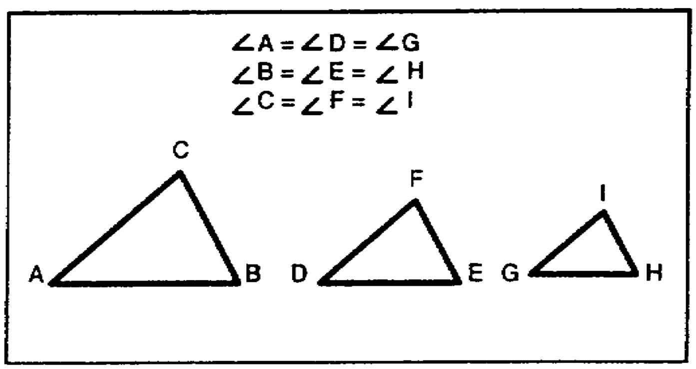


Figure 3-34. Equal angles

(1) Two triangles are similar if two angles of one triangle are equal to the two corresponding angles of the other triangle.

(2) Two right angles are similar if an acute angle of one triangle is equal to an acute angle of the other triangle.

c. Theorem 3. Two triangles are similar if their sides are respectively parallel or if their sides are respectively perpendicular. When this condition exists, the corresponding angles of the two triangles are similar.

d. Theorem 4. In a right triangle, if a perpendicular line is drawn from the vertex of the right angle to the hypotenuse, the resulting two triangles are similar to the original triangle, and they are similar to each other. This relationship is shown in Figure 3-35, page 3-20.

e. Theorem 5. In a right triangle, if a perpendicular line is drawn from the vertex of the right angle to the hypotenuse, the perpendicular line is the mean proportion between the segments of the hypotenuse. The following proportion can then be written for the right triangle shown in Figure 3-35:

AD : CD = CD : DB

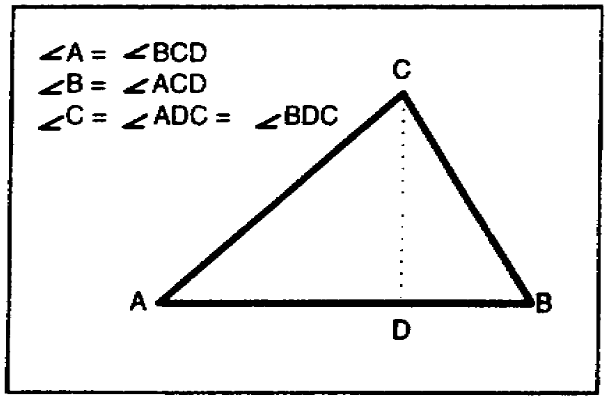


Figure 3-35. Right angles

3-16. Theorem for the Area of a Triangle. The area of a triangle is equal to one-half the product of its base and its altitude. This theorem can be proved with the aid of Figure 3-36, which shows a parallelogram (ABCD) whose sides are parallel to two sides of triangle ABC. Triangles ABC and ACD are equal; therefore, the area of triangle ABC is one-half the area of parallelogram ABCD. The area of parallelogram ABCD is the product of triangle ABC's base and altitude.

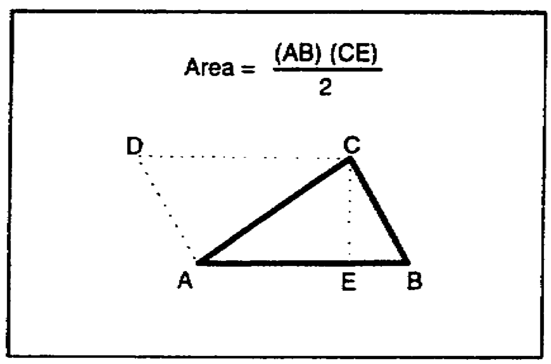


Figure 3-36. Area of a triangle

3-17. Equating like quantities. By using the theorems for similar triangles, you can often determine distances that cannot be readily measured directly. For example, the antenna reflector in Figure 3-37 casts a shadow on the ground that is 35 feet 9 inches from a point directly beneath the antenna pole. At the same time, a yardstick in a vertical position casts a shadow that is 1 foot 2 inches in length. Since light from the sun strikes both the antenna pole and the yardstick at the same angle, angles C and C^1 are equal, and triangles ABC and $A^1B^1C^1$ are similar. Using the skills of equating like qualities that you learned in Lesson 2 under Ratios and Proportions, set up the problem as follows:

$$\frac{h}{s} = \frac{h^1}{s^1}$$

Then, substituting known values and solving for h:

$$\frac{h}{35.75} = \frac{3 \times 12}{(12+2)}$$

$$h = \frac{35.75 \times 36}{14}$$

$$h = 92 \text{ feet}$$

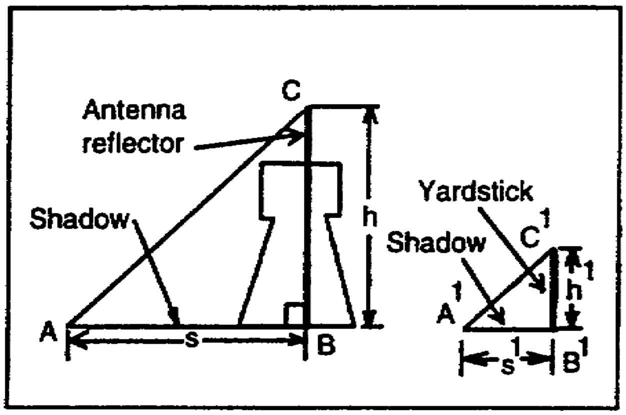


Figure 3-37. Height of antenna reflector

PART F - REVIEW OF GEOMETRY

3-18. Reviewing Geometric Principles. While the actual use of geometry in solving surveying problems is somewhat limited, many of its terms, postulates, theorems, and corollaries have an important bearing on trigonometric computation. The triangle, as you know, plays a very important part in the profession of surveying. For that reason, the following paragraphs will reference you back to pertinent portions of the text in this lesson.

a. In paragraph 3-1, various terms useful in solving geometric problems were defined. Among these were a point, lines (both straight and curved), and flat surfaces. Careful analysis showed that both lines and surfaces were actually generated by a point moving in a straight or a constantly changing direction. In one sense, a line could be considered as the edge view of a flat surface; thus, a straight line could very well be one edge of a flat plane while a curved line could represent one edge of a curved surface.

In plane geometry, a flat plane can be developed by moving a straight line in a direction 90° to its length or by fixing one end of the straight line and swinging its free end a designated distance. By such controlled movement, a segment of a circle that closely resembles a triangle or a complete circle (both of which are flat planes) can be generated from a straight line. The distance that such a straight line is allowed to swing in an arc is measured in special units which are known as degrees, minutes, or seconds. Each of these measurement units is useful when measuring or determining the size of an angle.

b. An angle is ordinarily thought of as being formed by the junction of two straight lines that may or may not continue beyond the junction. If the lines do continue, then the two opposite angles at the vertex are known as vertical angles. This situation was illustrated in Figure 3-7. When the two lines intersect each other at 90°, then four equal angles are formed and are known as right angles. The two right angles sharing a common side and vertex are known as supplementary angles. The opposite or exterior sides of the two right angles form what is known as a straight angle. Each pair of right angles, sharing a common side, can also be called adjacent angles.

c. To solve many of the geometric problems, certain principles have to be taken as they stand and without proof . Based on personal observation, these principles or postulates should be firmly fixed in your mind. There are 25 postulates in all, and they are listed in paragraph 3-6. To aid you in analyzing geometric problems, you can use the superposition method, which allows you to compare by superposition two lines, angles, or figures to determine whether they are equal or congruent.

d. Solving geometric problems is largely based on using theorems and corollaries. The latter term refers to a secondary rule whose truth can be readily derived from a theorem. For convenience, the theorems that are commonly used in plane geometry have been grouped together. Thus, the previous text includes eight theorems which are essential to solving problems that involve lines and angles. These eight theorems are listed under paragraphs 3-9 and 3-10; other theorems that aid in solving triangles are given in paragraphs 3-11 through 3-16.

PRACTICE EXERCISE

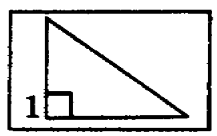
The following items will test your grasp of the material covered in this lesson. When you have completed the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that par which contains the potion involved.

1. If side A of a triangle is 18 units, side B is 24 units, and side C is 30 units, this triangle is referred to as a

   A. 95 unit triangle
   B. Base 5 triangle
   C. Hypotenuse triangle
   D. 3-4-5 triangle

2. **The small ┐ at point 1 is called __________.**

   A. Point of intersection
   B. Angle symbol
   C. Right-angle symbol
   D. Square unit symbol



3. _________ is generated by a point in motion.

   A. A plane
   B. A line
   C. A angle
   D. A surface

4. Fundamental principles of geometry that depend on personal observation and must be taken without proof are called

   A. Postulates
   B. Theorems
   C. Theories
   D. Assumptions

5. Lines that bound a triangle are called

   A. Sides
   B. Interior and exterior angles
   C. Bisectors
   D. Reentrant angles

LESSON 3

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct answer and feedback	
1.	D	3-4-5 triangle A special case of...(page 3-18, para 3-14b)
2.	C	Right-angle symbol The small square at...(page 3-3, para 3-2a and Figure 3-4)
3.	B	A line A line is generated by...(page 3-1, para 3-1)
4.	A	Postulates Fundamental principles of geometry...(page 3-7, para 3-6)
5.	A	Sides The lines that bound the...(page 3-5, para 3-5)

LESSON 4

TRIGONOMETRY

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn how to use trigonometry.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will be taught how to use trigonometry in surveying operations.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, CDC 3E551A, and Appendix C of this ACCP.

INTRODUCTION

Surveyors work almost daily with the triangle in surveying vast expanses of land. In doing so, they use a network of triangles, which is known as triangulation. The sides and angles of a plane triangle are so related that given any three parts, provided at least one of them is a side, the shape and size of a triangle can be determined. This science, which is called trigonometry, is both geometric and algebraic in nature. This branch of mathematics deals with computing unknown parts. It begins by showing the mutual dependence of the sides and angles in a triangle and, for that purpose, employs the ratio of the sides in a right triangle. It is based on the properties of similar triangles and is applied whenever angles enter into the solution of the problem. Because trigonometry deals primarily with angles, it is necessary for the surveyor to have a clear conception of the meaning and the measurement of angles.

PART A - ANGLES

4-1. Defining Angles. An angle is defined as the figure formed by the intersection of two lines at one point. The point is called the vertex of the angle. The two lines forming the angle are called the sides or legs of the angle. The angle, as it applies to trigonometry, is read by designating the capital letter placed at the vertex. The mathematical symbol for the word angle is simply a small $\angle$. The size of magnitude of an angle is determined by the difference in direction of the two sides. This size is measured in degrees (°), minutes ('), and seconds (").

a. The minutes are subdivisions of a degree. There are 60' in 1°. The seconds are a subdivision of the minutes. There are 60" in 1' and 3,600" in 1°. Seconds are usually subdivided into tenths and hundredths of a second and are always expressed as decimal fractions of a second.

b. The basis of all angles is the circle, which contains 360°. The four ways 360° may be written are as follows:

- 360°
- 360° 00' 00"
- 359° 59' 60"
- 0° 00' 00"

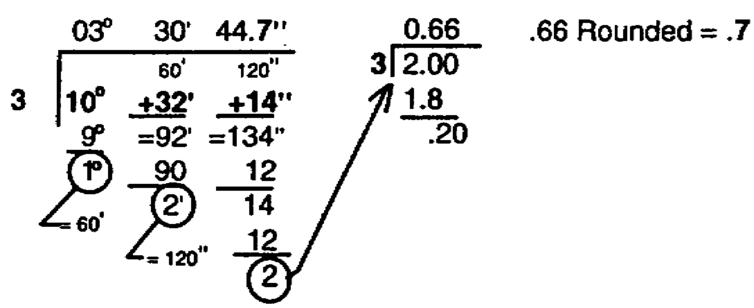
It should always be remembered that each of the minutes and seconds columns must have at least two figures; therefore, many times a zero must be added in front for a single digit or two zeros must be added to denote no minutes or seconds.

4-2. Computing Angles. The division of an angle is done by dividing the number into degrees, minutes, and seconds. The important point to remember in the first portion of this process is that 1° equals 60' and when there are degrees remaining after the initial division, they must be converted to minutes before carrying them over to the minutes column. As shown in the example below, 3 will go into 10° 3 times, leaving a remainder of 1°. Before making the second division, this 1° must be changed to 60' and added to the number of minutes in the original problem.

a. The original problem shows 32'. Add 60' (the remainder of 1°) to 32', which results in a sum of 92. Divide 92 by 3, which will go 30 times with a remainder of 2'.

Example: Divide 10° 32' 14" by 3.

Solution:



b. Since there are 60" in 1 minute, this same process must be repeated for any remaining minutes after the second division. Therefore, change the 2' remainder to 120" and add it to 14" (the seconds shown in the original problem), which results in a sum of 134". Divide 134" by 3, which will go 44 times with a remainder of 2". By converting this remainder to a decimal fraction (2/3 of a minute equals 0.67 or rounding off to 0.7), you arrive at the answer desired. If there are no degrees or minutes to be carried over, straight division is performed.

c. There will be times when the same angle must be used in field computations a number of times. The simplest way to multiply an angle by a number is to perform each of the following steps:

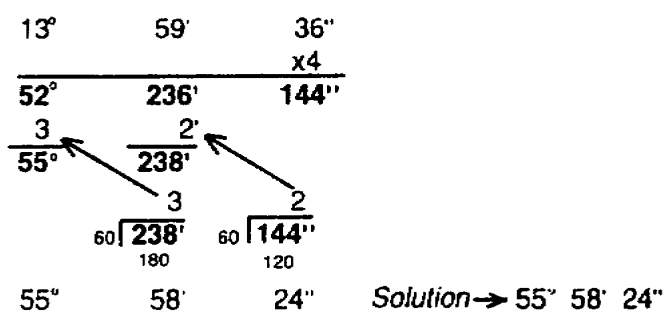
- Multiply the seconds by the number to determine the total number of seconds.

- Multiply the minutes by the number to determine the total number of minutes.

- Multiply the degrees by the number to determine the total number of degrees.

Example: Multiply 13° 59' 36" by 4.

Solution:



d. If there are more than 59 seconds in the total seconds, the minutes must be extracted from the seconds. This is done by dividing 60, the number of seconds in 1 minute, into the total number of seconds. The quotient represents the number of minutes contained in the seconds, and the remainder represents the odd number of seconds. This process is repeated to extract the degrees from the total minutes if there are more than 59 after adding the minutes found in the seconds column. If there are any degrees contained in the minutes after they have been extracted, they are added to the total amount of degrees previously determined.

4-3. Converting Angles. Converting degrees, minutes, and seconds to degrees and decimal parts of a degree is done in the following manner:

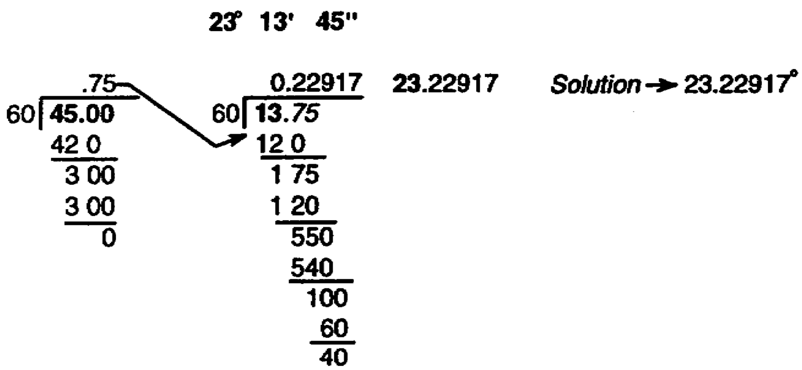
- Divide the seconds by 60 to convert them to decimal parts of a minute.

- Add the quotient to the minutes, and divide this sum by 60 to convert it to decimal parts of a degree.

- Add the quotient to the degrees; the sum being the converted form of degrees, minutes, and seconds.

Example: Convert 23° 13' 45" to a decimal fraction.

Solution:



a. The conversion of degrees and decimal parts of a degree to minutes and seconds is done in the following manner:

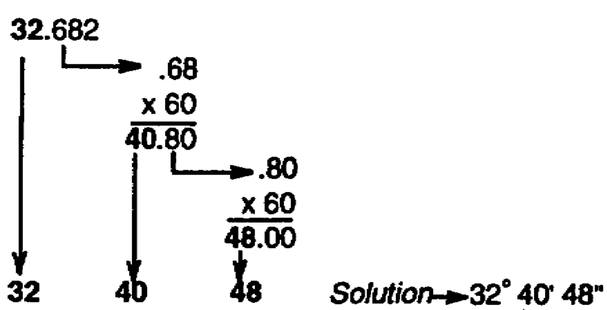
- Multiply the decimal by 60 to convert it to minutes and decimal parts of a minute.

- Multiply the decimal par of a minute by 60 to convert it to seconds.

The result obtained is degrees, minutes, and seconds.

Example: Convert 32.682 to degrees, minutes, and seconds.

Solution:



b. As you will discover, there will be many occasions when two or more angles must be added together. The addition of two or more angles is done in the following manner.

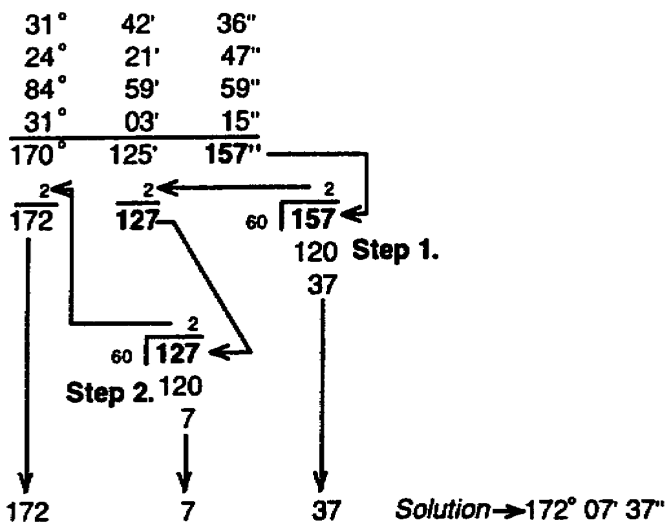
- Add the seconds together.

- Add the minutes together.

- Add the degrees together.

Example: Add the following.

Solution:



c. After the answer has been reached, the minutes must be extracted from the seconds and the degrees must be extracted from the minutes (if there are any) by dividing by 60.

d. The mean angle of two or more angles is determined by combining the processes of addition and division in the following order:

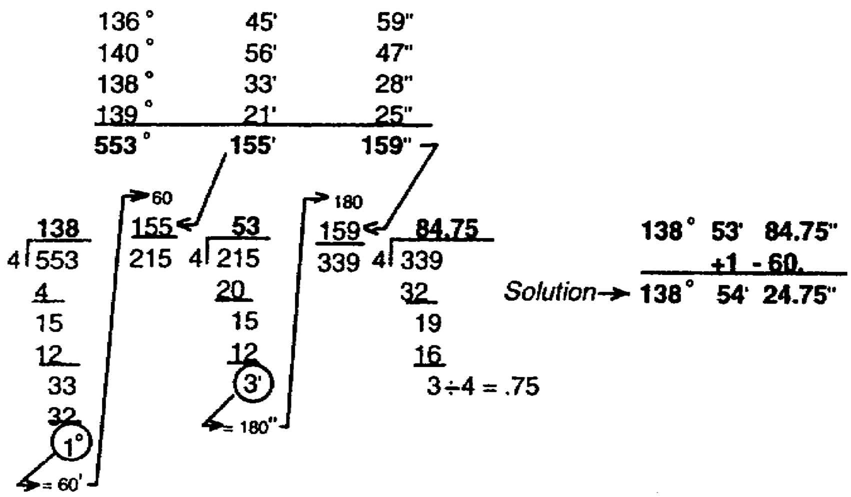
- Add all the degrees, minutes, and seconds.

- Divide the sum by the number of angles.

The quotient is the mean angle.

Example: Find the mean angle.

Solution:



e. Accurately subtracting two or more angles is an important step in computing the length of the forward and back azimuths of a geodetic line based on the known geodetic positions of the ends of the line. This process is known as inverse-position computation. Here are some precautions to observe.

(1) If the minuend is larger than the subtrahend, subtraction is performed. But keep in mind when borrowing degrees or minutes that there are 60' in 1° and 60" in 1', and they must be carried over in this manner.

Example: Subtract 254° 15' 10" from 435° 20' 59".

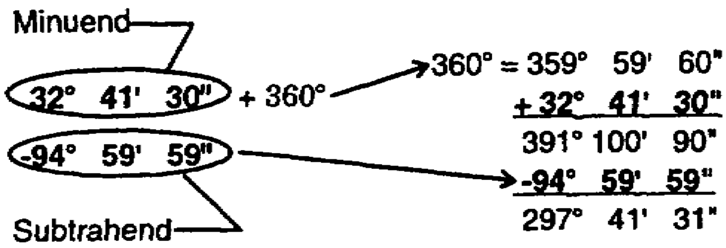
Solution:

435° 20' 59"
-254° 15' 10"
181° 05' 49"

(2) If the minuend is smaller than the subtrahend, 360° must be added to the minuend. This may be done without actually changing the value of the angle, as may be illustrated by an angle formed by two radii of a circle.

Example: Subtract 94° 59' 59" from 32° 41' 30".

Solution:



f. If one radius is held stationary and the other is moved 360°, it will return to its original direction without increasing or decreasing the magnitude of the angle. This principle, after adding 360° to the minuend, will allow you to perform straight subtraction.

PART B - TRIGONOMETRIC FUNDAMENTALS

4-4. Functions of Acute Angles. Understanding trigonometric fundamentals is essential to studying surveying, map making, map reading, astronomy, navigation, and many other engineering subjects where objects are represented by drawn figures or are given exact locations. Trigonometry primarily deals with measuring angles and distances. As a surveyor, you will be apply the principles for solving a right triangle to obtain the indirect measurement of angles and distances.

a. In Figure 4-1, note that the triangle's angles are identified by capital letters, and the sides opposite each angle are identified by the same letter only in small print. A general method of identifying the sides of a right triangle is by giving them names in reference to the angles. Referring to the acute angle A in Figure 4-1, side a is known as the side opposite and side b is known as the side adjacent. With reference to the acute angle B, side b is the side opposite and side a the side adjacent. The side opposite the right angle (c in this case) is always known as the hypotenuse.

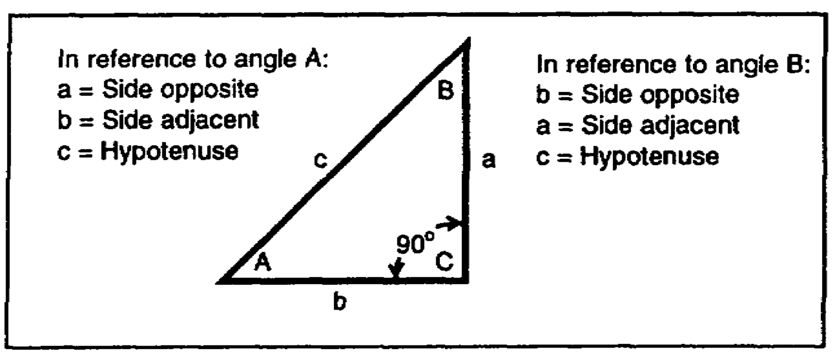


Figure 4-1. Triangle relationships

b. Referring to Figure 4-1, you will see that the following six ratios may be written for the sides of a right triangle: $\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{c}{a}, \frac{c}{b}$, and $\frac{b}{a}$. In Figure 4-2 the sides are extended to produce the triangle AB^1C^1 with sides $a^1b^1c^1$. By using the rule of similar triangles, you may set up the following proportion for this triangle.

$$\frac{AB}{AC} = \frac{AB^1}{AC^1}$$

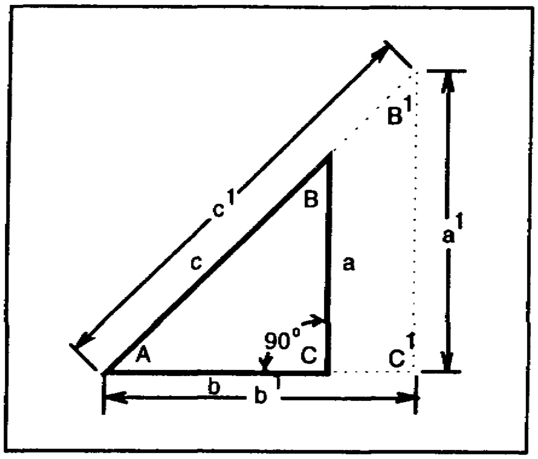


Figure 4-2. Similar triangles

c. Looking at the proportion, note that the difference in the triangle's size does not affect the ratio of the hypotenuse to the base. The ratio is the same for all right triangles with the same acute angle. In any right triangle, the ratio of any two sides depends only on the size of the related acute angle. Therefore, this rule can be applied to the six ratios written above; however, their values depend on the size of the angle.

d. To differentiate between and clearly identify each ratio, note the special names given to them in Table 4-1 (see Figure 4-2 to find the pertinent side and angle). In Table 4-1, the ratio $\frac{a}{c}$ is the sine of A and also the cosine of B. Similarly, $\frac{b}{c}$ is the cosine of A and the sine of B, and the same will hold true for all the other ratios. Therefore, you cannot learn these ratios as $\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{c}{a}, \frac{c}{b}$, or $\frac{b}{a}$, but they must be memorized as $\frac{\text{side adjacent}}{\text{hypotenuse}}$ and so on.

Table 4-1. Special names for ratios

Considering Angle A	Considering Angle B
$\frac{a}{c} = \frac{\text{side opposite}}{\text{hypotenuse}} = \sin A$	$\frac{a}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}} = \cos B$
$\frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}} = \cos A$	$\frac{b}{c} = \frac{\text{side opposite}}{\text{hypotenuse}} = \sin B$
$\frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}} = \tan A$	$\frac{a}{b} = \frac{\text{side adjacent}}{\text{side opposite}} = \cot B$
$\frac{c}{a} = \frac{\text{hypotenuse}}{\text{side opposite}} = \csc A$	$\frac{c}{a} = \frac{\text{hypotenuse}}{\text{side adjacent}} = \sec B$
$\frac{c}{b} = \frac{\text{hypotenuse}}{\text{side adjacent}} = \sec A$	$\frac{c}{b} = \frac{\text{hypotenuse}}{\text{side opposite}} = \csc B$
$\frac{b}{a} = \frac{\text{side adjacent}}{\text{side opposite}} = \cot A$	$\frac{b}{a} = \frac{\text{side opposite}}{\text{side adjacent}} = \tan B$
NOTE: **Sin = sine** **Cos = cosine** **Tan = tangent**	**Sec = secant** **Csc = cosecant** **Cot = cotangent**

4-5. Functions of Trigonometry. If A is any given angle, a set of values can be determined for the six ratios. Because they will vary with the changes in A, the sine A, cosine A, tangent A, cotangent A, secant A, and cosecant A are functions of A and are specifically referred to as trigonometric functions. See Appendix C for the Natural Trigonometric-Functions Tables.

a. In general, the decimal form of a function is an endless decimal. By the use of advanced mathematics, an angle's function can be computed to as many decimal places as desired. In any table of function values, the error in any entry is at most one-half of a unit in the last place. A natural trigonometric-functions table may show 4 to 10 place values and is easy to use. Because of the complementary relationship of the acute angles in a right triangle, that is, sine A = cosine (90-A) and so forth, each entry in a natural trigonometric-functions table serves a dual purpose and, consequently, the table is only one-half as large as it otherwise would be.

b. Figure 4-3, page 4-10, is an example of a natural trigonometric-functions table. From this table, find the sine 32° 20' using the procedures below:

- Locate the page with 32° at the top.
- Find the sin and cos columns under 32°.
- Locate the word minutes at the left of the page. Going down this column, go to 20'.
- Go right across this line to the sin column. There you will find 0.53484, which is the sine of 32° 20'.

	32 Degrees				
Minutes	Sin	Cos	Tan	Cot	
0	0.52992	0.84805	0.62487	1.60033	60
1	0.53017	0.84789	0.62527	1.59930	59
2	0.53041	0.84774	0.62568	1.59826	58
3	0.53066	0.84759	0.62608	1.59723	57
4	0.53091	0.84743	0.62649	1.59620	56
5	0.53115	0.84728	0.62689	1.59517	55
6	0.53140	0.84712	0.62730	1.59414	54
7	0.53164	0.84697	0.62770	1.59311	53
8	0.53189	0.84681	0.62811	1.59208	52
9	0.53214	0.84666	0.62852	1.59105	51
10	0.53238	0.84650	0.62892	1.59002	50
11	0.53263	0.84635	0.62933	1.58900	49
12	0.53288	0.84619	0.62973	1.58797	48
13	0.53312	0.84604	0.63014	1.58695	47
14	0.53337	0.84588	0.63055	1.58593	46
15	0.53361	0.84573	0.63095	1.58490	45
16	0.53386	0.84557	0.63136	1.58388	44
17	0.53411	0.84542	0.63177	1.58286	43
18	0.53435	0.84526	0.63217	1.58184	42
19	0.53460	0.84511	0.63258	1.58083	41
20	0.53484	0.84495	0.63299	1.57981	40
21	0.53509	0.84480	0.63340	1.57879	39
22	0.53534	0.84464	0.63380	1.57778	38
23	0.53558	0.84448	0.63421	1.57676	37
24	0.53583	0.84433	0.63462	1.57575	36
25	0.53607	0.84417	0.63503	1.57474	35
26	0.53632	0.84402	0.63544	1.57372	34
27	0.53656	0.84386	0.63584	1.57271	33
28	0.53681	0.84370	0.63625	1.57170	32
29	0.53705	0.84355	0.63666	1.57069	31
30	0.53730	0.84339	0.63707	1.56969	30
31	0.53754	0.84324	0.63748	1.56868	29
32	0.53779	0.84308	0.63789	1.56767	28
33	0.53804	0.84292	0.63830	1.56667	27
34	0.53828	0.84277	0.63871	1.56566	26
35	0.53853	0.84261	0.63912	1.56466	25
36	0.53877	0.84245	0.63953	1.56366	24
37	0.53902	0.84230	0.63994	1.56265	23
38	0.53926	0.84214	0.64035	1.56165	22
39	0.53951	0.84198	0.64076	1.56065	21
40	0.53975	0.84182	0.64117	1.55966	20
41	0.54000	0.84167	0.64158	1.55866	19
42	0.54024	0.84151	0.64199	1.55766	18
43	0.54049	0.84135	0.64240	1.55666	17
44	0.54073	0.84120	0.64281	1.55567	16
45	0.54097	0.84104	0.64322	1.55467	15
46	0.54122	0.84088	0.64363	1.55368	14
47	0.54146	0.84072	0.64404	1.55269	13
48	0.54171	0.84057	0.64446	1.55170	12
49	0.54195	0.84041	0.64487	1.55071	11
50	0.54220	0.84025	0.64528	1.54972	10
51	0.54244	0.84009	0.64569	1.54873	9
52	0.54269	0.83994	0.64610	1.54774	8
53	0.54293	0.83978	0.64652	1.54675	7
54	0.54317	0.83962	0.64693	1.54576	6
55	0.54342	0.83946	0.64734	1.54478	5
56	0.54366	0.83930	0.64775	1.54379	4
57	0.54391	0.83915	0.64817	1.54281	3
58	0.54415	0.83899	0.64858	1.54183	2
59	0.54440	0.83883	0.64899	1.54085	1
60	0.54464	0.83867	0.64941	1.53986	0
	Cos	Sin	Cot	Tan	Minutes
			57 Degrees		

Figure 4-3. Example of a natural trigonometric-functions table

c. To read the sine or cosine of an angle greater than 45°, it is necessary to look at the bottom of the page reading up instead of down. The column for minutes will be on the right instead of the left. To find the cosine of 57° 18', see Figure 4-3. The cosine of 57° 18' is 0.84151. To find the tangant and cotangant of an angle, use the same procedure.

d. Often you will be given the sides of a right triangle, and you will want to find the acute angles of it. Remember that the sine of an angle is equal to the opposite side divided by the hypotenuse. By making a fraction of the two sides' values, you will get the value of the sine function of the angle. To find the angle, look up this value in the proper table under the sin column and then read the number of degrees and minutes in a manner opposite to that used when finding a function for a given angle. For example, if by dividing

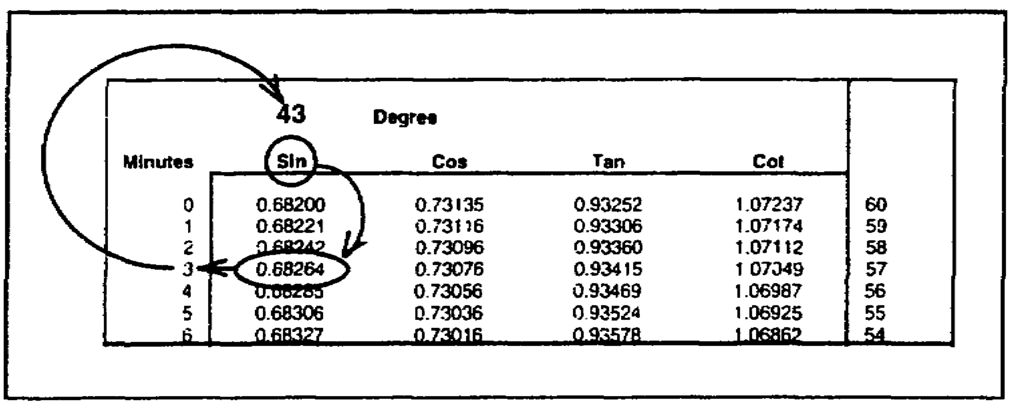
the opposite side by the hypotenuse you find the sine value to be $\frac{1.36528}{2}$**, or 0.68264, do**

the following:

- Locate this number under the sin column in Figure 4-4. You find your angle to be 43°.

- Note that the angle is on the top of the page; therefore, the minute portion of your angle will be found in the minutes column on the left-hand side of the page. Directly on the line to the left of 0.68264 in the minutes column, you find 03'. The final answer then would be 43° 03'. This problem would be written in the following manner:

sin A = 0.68264
A = 43° 03'



	43 Degrees				
Minutes	Sin	Cos	Tan	Cot	
0	0.68200	0.73135	0.93252	1.07237	60
1	0.68221	0.73116	0.93306	1.07174	59
2	0.68242	0.73096	0.93360	1.07112	58
3	0.68264	0.73076	0.93415	1.07049	57
4	0.68285	0.73056	0.93469	1.06987	56
5	0.68306	0.73036	0.93524	1.06925	55
6	0.68327	0.73016	0.93578	1.06862	54

Figure 4-4. Finding the acute angle

e. So far you have learned how to find the functions of angles expressed in degrees and minutes. However, it is sometimes necessary to find the functions of angles expressed in seconds. This is done by a process called interpolation. For every change of degrees, minutes, or seconds in an angle, there is a proportional change in the function of the angle. By the use of this proportion, it is possible to interpolate for the seconds of an angle.

f. Suppose you wanted to find the sine 21° 12' 20". Since Figure 4-5 shows only the degrees and minutes, you must interpolate for the seconds. Notice that 21° 12' 20" is $\frac{20}{60}$ or one-third of the way from 21° 12' 00" to 21° 13' 00", whose functions can be obtained from Figure 4-5. By the principle of proportional parts discussed in the previous paragraph, an increase of 20" in the angle 21° 12' causes one-third as much change in its sine as is caused by an increase of 1 minute in the angle.

	21 Degree				
Minutes	Sin	Cos	Tan	Cot	
0	0.35837	0.93358	0.38386	2.60509	60
1	0.35864	0.93348	0.38420	2.60283	59
2	0.35891	0.93337	0.38453	2.60057	58
3	0.35918	0.93327	0.38487	2.59831	57
4	0.35945	0.93316	0.38520	2.59606	56
5	0.35973	0.93306	0.38553	2.59381	55
6	0.36000	0.93295	0.38587	2.59156	54
7	0.36027	0.93285	0.38620	2.58932	53
8	0.36054	0.93274	0.38654	2.58708	52
9	0.36081	0.93264	0.38687	2.58484	51
10	0.36108	0.93253	0.38721	2.58261	50
11	0.36135	0.93243	0.38754	2.58038	49
12	0.36162	0.93232	0.38787	2.57815	48
13	0.36190	0.93222	0.38821	2.57593	47
14	0.36217	0.93211	0.38854	2.57371	46
15	0.36244	0.93201	0.38888	2.57150	45
16	0.36271	0.93190	0.38921	2.56928	44
17	0.36298	0.93180	0.38955	2.56707	43

Figure 4-5. Interpolation

Example: Find the value of sine 21° 12' 20". The stated value of any angle is known as its tabular reading.

Solutions:

First Method: Set up the ratio.

sin 21° 13'= 0.36190
sin 21° 12'= <u>0.36162</u>
0.00028 difference

So: $\frac{x}{28} :: \frac{20"}{60"}$ or x = 9

Therefore: sin 21° 12'= 0.36162

By interpolation: 20"= 9

Then: sin 21° 12' 20" = 0.36171

Second Method: Take 1/3 tabular difference.

sin 21° 13' = 0.36190
sin 21° 12' = 0.36162
0.00028 difference

So: 1/3 (0.00028) = 0.00009

Therefore: sin 21° 12' = 0.36162
1/3 tabular difference = 0.00009
Then: sin 21° 12' 20" = 0.36171

g. It must be remembered that the tabular values of the sine and tangent increase as the size of the angle increases, and the tabular values of the cosine and cotangent decrease as the size of the angle increases. To avoid confusion and errors in interpolation, it is advisable to follow the methods used above, adding the proportional parts for seconds in the case of sines and tangents and subtracting the proportional parts in the case of cosines and cotangents.

h. As you will recall, the triangle has six parts: three angles (one of which is 90°) and three sides (see Figure 4-6, page 4-14). If two sides, or one side and an acute angle, are given, you can compute the unknown parts of the triangle. This computation is called the solution of a triangle. From this statement, it is evident that in order to solve a right triangle, two parts besides the right angle must be given, at least one of them being a side. The two given parts may be-

- An acute angle and the hypotenuse.
- An acute angle and the opposite leg.
- An acute angle and the adjacent leg.
- The hypotenuse and a leg.
- The two legs.

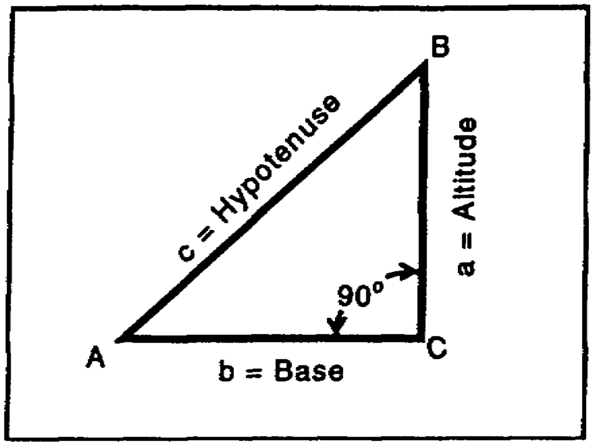


Figure 4-6. A right triangle

i. Before any attempt is made to solve a trigonometric problem, the following steps should be completed.

- Construct a figure as near as possible to scale.
- Letter each given part on the diagram.
- Outline the solution, specifying each triangle and formula to be used.
- Solve each formula for the unknown quantity.

j. To solve the right triangle shown in Figure 4-7, you must find all six parts by following the steps above.

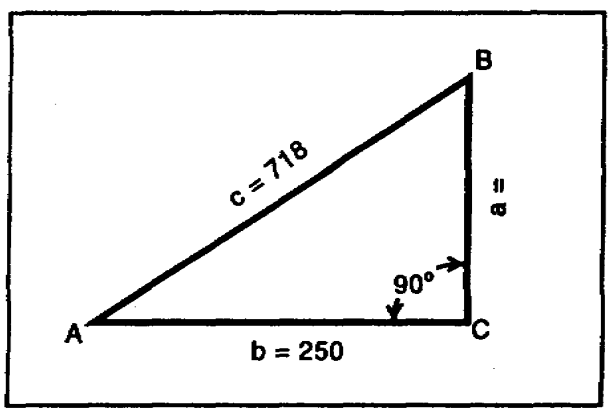


Figure 4-7. Sketch of problem

Example: Solve the right triangle.

Solution:

Known Quantities	*Unknown Quantities*	*Formulas to be used*
b = 250	∠B	$B = 90° - A$
c = 718	∠A	$\cos A = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}}$
	side a	$\tan A = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$

Formula for checking: a = c cos B

Solution:
Determine A: $\cos A = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{250}{718} = 0.34819$

60" (x)	cos 69° 37' 00" =	0.34830
	69° 37' ?	cos A = 0.34819 (11) (27)
	cos 69° 38' 00" =	0.34803

Taken from Figure 4-8

$\frac{x}{60} = \frac{11}{27}$ = 11 divided by 27 = .4074 multiplied by 60 = 24.4 or 24

then: A = 69° 37' 24"

Determine B:
B = 90° - A
90 = 89° 59' 60"
69° 37' 24"
20° 22' 36"

Then: B = 20° 22' 36"

Determine a:
a = b tan A =
250 tan 69° 37' 24"

60" (24)	tan 69° 37' 00" =	2.69131 (x) (.0024)
	tan 69° 37' 24" =	?
	tan 69° 38' 00" =	2.69371

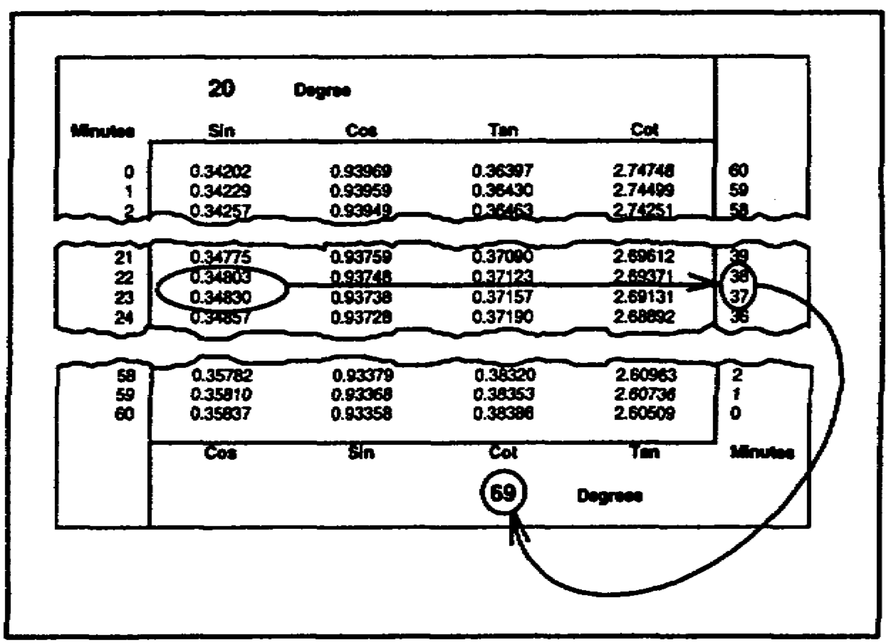
Taken from Figure 4-9, page 4-16

$\frac{24}{60} = \frac{x}{.0024}$ = 24 divided by 60 = .4 multiplied by .0024 = .00096

2.69131
.00096
2.69227

a = b tan A = 250 multiplied by 2.69227 = 673.067

Then a = 673.07



20 Degree					
Minutes	Sin	Cos	Tan	Cot	
0	0.34202	0.93969	0.36397	2.74748	60
1	0.34229	0.93959	0.36430	2.74499	59
2	0.34257	0.93949	0.36463	2.74251	58
21	0.34775	0.93759	0.37090	2.69612	39
22	0.34803	0.93748	0.37123	2.69371	38
23	0.34830	0.93738	0.37157	2.69131	37
24	0.34857	0.93728	0.37190	2.68892	36
58	0.35782	0.93379	0.38320	2.60963	2
59	0.35810	0.93368	0.38353	2.60736	1
60	0.35837	0.93358	0.38386	2.60509	0
	Cos	Sin	Cot	Tan	Minutes
			69 Degrees		

Figure 48. Sines and cosines

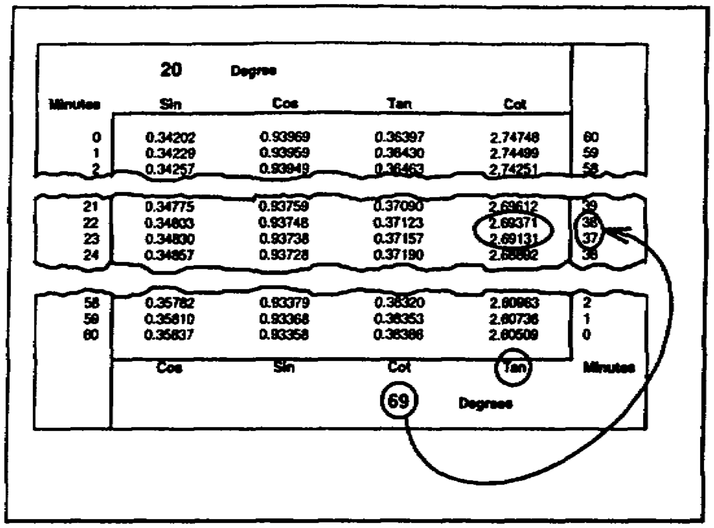


Figure 4-9. Tangents and cotangents

4-6. Functions of Obtuse Angles. Previously, you studied the functions of the acute angles of a right triangle. As a surveyor in the field, you are often required to turn oblique angles, that is angles greater than 90°. It is necessary for you to know the functions of angles greater than 90° in order to solve an oblique triangle. If one of the angles of a triangle exceeds 90°, you no longer have a right triangle. It is possible to express the functions of angles greater than 90° as ratios of the sides of right triangles, and relations can be derived between such functions and the functions of acute angles.

a. To determine the functions of angles greater than 90°, you will construct a pair of straight lines intersecting at right triangles, as shown in Figure 4-10. The intersection point of the two lines is called the origin and is labeled A. The horizontal line is labeled XX^1, and the perpendicular line is labeled YY^1. Using these lines as axes of a rectangular coordinate system, a point (such as B) can be located by its coordinates x and y. The x coordinate is positive when B is to the right of YY^1 and negative when B is to the left of YY^1. The y coordinate is positive when B is above XX^1 and negative when B is below XX^1. Using the origin as center and any convenient radius, you can construct a circle that will enable you to determine the functions of angles from 0° through 360°. This is shown in Figure 4-11.

b. Referring to Figure 4-11, you see that the circle is divided into four equal parts, each containing 90° of arc. These parts are called quadrants and are labeled counterclockwise from the line AX as follows:

- The first quadrant is contained in the segment of the circle XAY and is lettered with the Roman numeral I. It contains the angles between 0° and 90°.

- The second quadrant is contained in the segment of the circle YAXP and is labeled II. It contains the angles between 90° and 180°.

- The third quadrant is contained in the segment of the circle X^1AY^1 and is lettered III. It contains the angles between 180° and 270°.

- The fourth quadrant is contained in the segment of the circle Y^1AX and is lettered IV. It contains the angles between 270° and 360°.

c. If you begin at the line AX and rotate a radius through 360°, you can construct any number of right triangles in each of the four quadrants shown in Figure 4-11. The radius will represent the hypotenuse of the right triangle, a perpendicular line from the intersection of the radius and the circle to

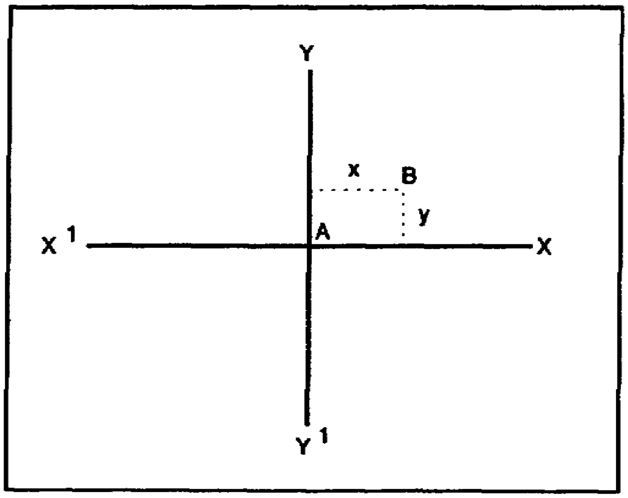


Figure 4-10. The coordinate system

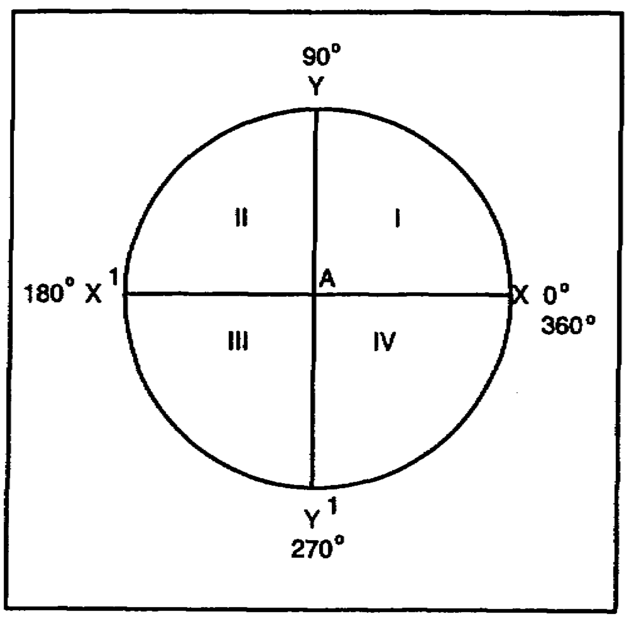


Figure 4-11. Quadrants of a circle

the line X^1X will be the side opposite the angle, and the distance from the center of the circle to where the perpendicular line intersects the line X^1X will be the side adjacent. Note that as the angle increases, the side opposite the side adjacent decreases until at 90° the side opposite is a radius and coincides with the line AY. As the angle increases from 90° to 180°, the side opposite will decrease and the side adjacent becomes a radius and the side opposite is 0°. From 180° to 270°, the side opposite increases and the side adjacent decreases until at 270° the side opposite is again a radius of the circle and the side adjacent is 0°. From 270° to 360°, the side opposite decreases and the side adjacent increases until at 360° the two sides are in the position of an angle of 0°. From the above, you can see that the side opposite can be expressed as a y distance and the side adjacent as an x distance.

d. Since the side opposite is essentially a y distance, you can affix positive signs to the sides opposite of any angle in this circle that lies above the horizontal line X^1X. Similarly, the negative sign can be affixed if the side opposite lies below the line X^1X. Likewise, if an angle has its side adjacent on the right side of the line Y^1Y, the side adjacent will be positive; if it lies on the left side of this line, it will be negative. Applying the above to Figure 4-11, you see that the angles from 0° to 90° have their side opposite above line X^1X and their side adjacent to the right of line Y^1Y. Therefore, all angles in the first quadrant have positive signs affixed to their sides opposite and adjacent. For angles in the second quadrant (90° - 180°, the side opposite is still above the line X^1X, but the side adjacent is to the left of line Y^1Y. Angles in quadrant II have positive sides opposite and negative sides adjacent. In the third quadrant (180° - 270°), the side opposite is below the line X^1X, and the side adjacent is still to the left of line Y^1Y. These angles have negative sides opposite and adjacent. For angles in quadrant IV (270° - 360°, the side opposite is still below the line X^1X, but the side adjacent is again to the right of line Y^1Y. Therefore, the side opposite is negative, and the side adjacent is positive. The hypotenuse of any right triangle is always considered to be positive.

e. It is always possible to express any one of the six trigonometric functions of any angle as a plus or minus trigonometric function of a positive angle less than 90°. Consider the problem of expressing the functions of 220° in terms of functions of an angle less than 90°. Referring to Figure 4-12, observe the following:

$$\sin 220° = \frac{y\,(\text{of } P)}{r} = \frac{-y}{r} = \frac{-y}{r}$$

$$\sin 40° = \frac{y}{r}$$

$$\therefore \sin 220° = -\sin 40°$$

f. The functions of any angle in the second quadrant (90° - 180°) can be found by using the above principles combined with the principles of supplementary angles.

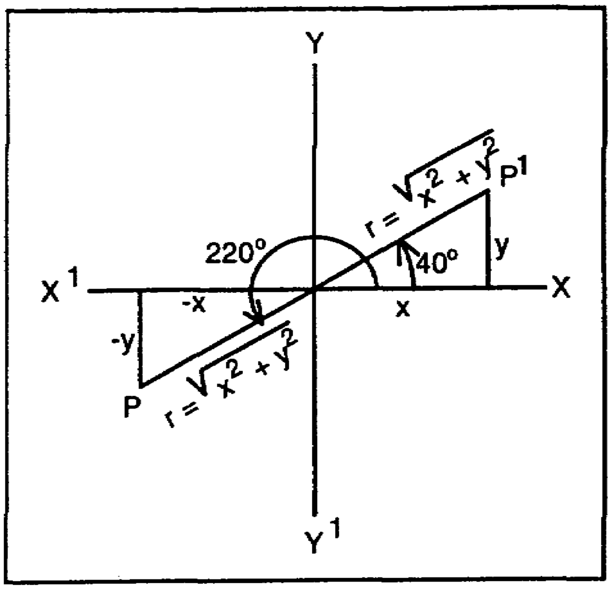


Figure 4-12. Trigonometric function of any angle

g. A supplementary angle is an angle that when added to another angle equals 180°. Since all angles in the second quadrant are larger than 90°, their supplements must be acute angles. Assuming that angle XAB in Figure 4-13, page 20, can have any value from 90° to 180°, you can derive the following formulas for the functions of any angle in the second quadrant.

(1) **Sin of angle XAB:**

$$\sin XAB = \frac{y}{r} = \sin X^1AB$$

$$\therefore \sin XAB = \sin (180° - XAB)$$

(2) **Cos of angle XAB:**

$$\cos XAB = \frac{-x}{r} = -\cos X^1AB$$

$$\therefore \cos XAB = -\cos (180° - XAB)$$

(3) **Tan of angle XAB:**

$$\tan XAB = \frac{y}{-x} = -\tan \frac{X}{AB}$$

$$\therefore \tan XAB = -\tan (180° - XAB)$$

(4) **Cot of angle XAB:**

$$\cot XAB = \frac{-x}{y} = -\cot X^1AB$$

$$\therefore \cot XAB = -\cot (180° - XAB)$$

(5) **Secant of angle XAB:**

$$\sec XAB = \frac{r}{-x} = -\sec X^1AB$$

$$\therefore \sec XAB = -\sec (180° - XAB)$$

(6) **Cosecant of angle XAB:**

$$\csc XAB = \frac{r}{y} = \csc X^1AB$$

$$\therefore \csc XAB = \csc (180° - XAB)$$

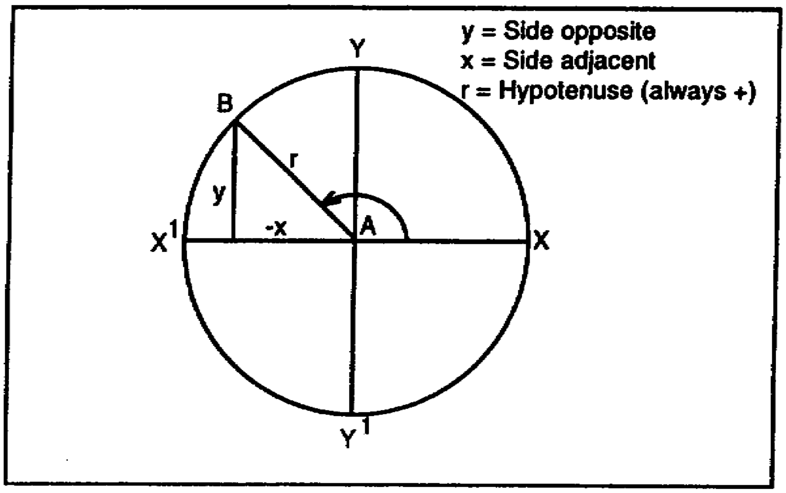


Figure 4-13. Angles between 90° and 180°

h. The functions of any angle between 180° and 270° can be expressed as the function of the acute angle, which is found by subtracting 180° from the given angle. Assuming that angle XAB in Figure 4-14 can have any value from 180° to 270°, you can derive the following formulas for the functions of any angle in the third quadrant.

(1) Sin of angle XAB:
$\sin XAB = -\sin X^1AB = -\sin (XAB - 180°)$

(2) Cos of angle XAB:
$\cos XAB = -\cos X^1AB = -\cos (XAB - 180°)$

(3) Tan of angle XAB:
$\tan XAB = \tan X^1AB = \tan (XAB - 180°)$

(4) Cot of angle XAB:
$\cot XAB = \cot X^1AB = \cot (XAB - 180°)$

(5) Sec of angle XAB:
$\sec XAB = -\sec X^1AB = -\sec (XAB - 180°)$

(6) Csc of Angle XAB:
$\csc XAB = -\csc X^1AB = -\csc (XAB - 180°)$

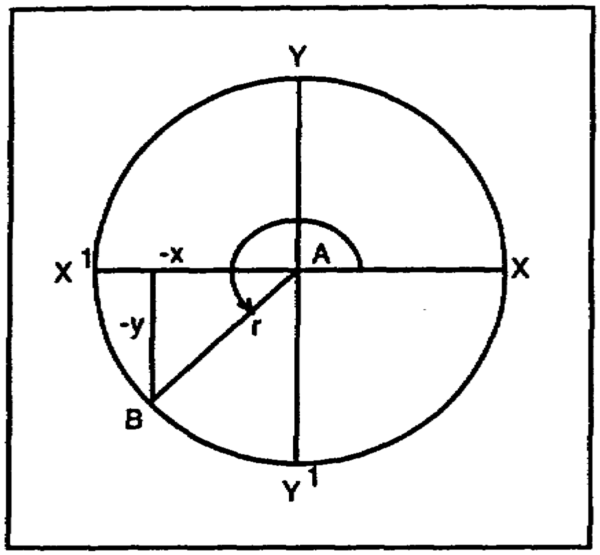


Figure 4-14. Angles between 180° and 270°

i. The functions of any angle between 270° and 360° may be expressed as the function of the acute angle, which is found by subtracting the given angle from 360°. Assuming that angle XAB in Figure 4-15, page 22, can have any value from 270° to 360°, you can derive the following formulas for the functions of any angle in the fourth quadrant.

(1) Sin of angle XAB:
$\sin XAB = -\sin (360° - XAB)$

(2) Cos of angle XAB:
$\cos XAB = \cos (360° - XAB)$

(3) Tan of angle XAB:
$\tan XAB = -\tan (360° - XAB)$

(4) Cot of angle XAB:
$\cot XAB = -\cot (360° - XAB)$

(5) Sec of angle XAB:
$\sec XAB = \sec (360° - XAB)$

(6) Csc of angle XAB:
$\csc XAB = -\csc (360° - XAB)$

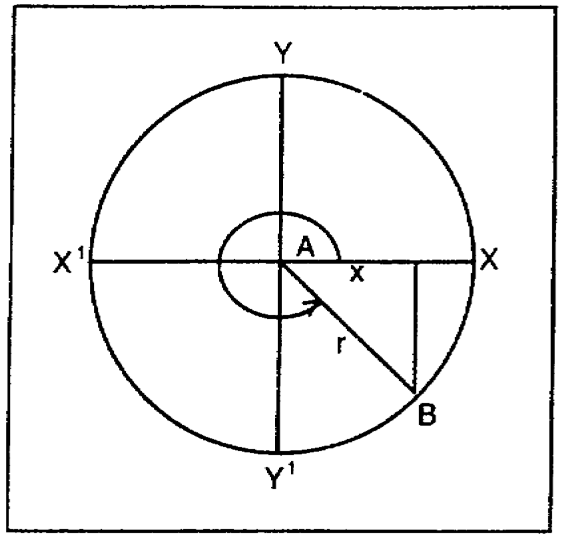


Figure 4-15. Angles between 270° and 360°

j. Table 4-2 shows the relationship of the sign, the function, and the quadrant of an angle.

Table 4-2. Relationship of the sign, the function, and the quadrant

Functions	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
Sine	+	+	-	-
Cosine	+	-	-	+
Tangent	+	-	+	-
Cotangent	+	-	+	-
Secant	+	-	-	+
Cosecant	+	+	-	-

4-7. The Unit Circle. If a circle has a radius of unity, then the numerical values of the functions for a given angle are represented by the lengths of the lines. This is illustrated for a second-quadrant angle in Figure 4-16.

4-8. Oblique Triangles. Many incidences arise in your daily work that deal with the solution of oblique triangles. To solve these triangles, the fundamental principles of the functions of angles greater than 90° must be understood. The functions of angles greater than 90° can be expressed as the functions of acute angles. These acute angles are found by subtracting the given angle from-

- 180° for quadrant II.
- 270° for quadrant III.
- 360° for quadrant IV.

To each of the functions obtained for these acute angles, you must affix the proper sign as determined from the size of the angle. Although there are numerous ways of numbering the quadrants of a circle, there will be no change in the signs of the functions of each quadrant as shown in Table 4-2.

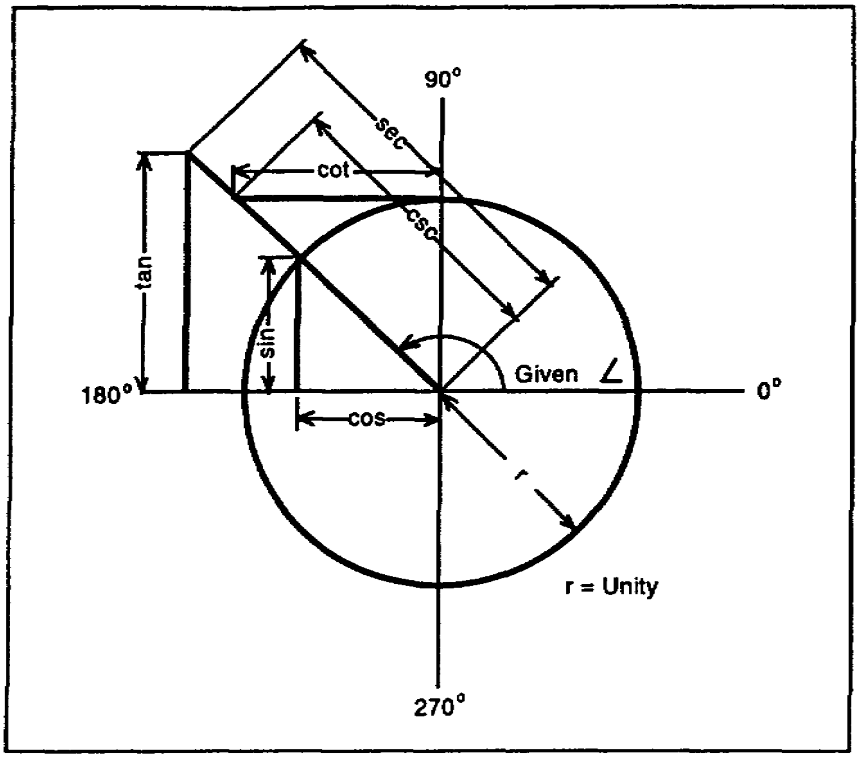


Figure 4-16. The unit circle

a. In previous paragraphs, you dealt with the solution of a special kind of triangle--the right triangle. Since the majority of the angles turned by the surveyor in the field are not right angles, you must know how to solve triangles that are not right triangles. In the following paragraphs, you will review the laws and the methods governing the solution of triangles.

b. It is possible to solve any triangle in which three of its six parts are known, providing one known part is a side. In a right triangle, the right angle is constant; so only a side and one other part must be known to solve the triangle. However, in the oblique triangle, one side and two other parts must be known before the triangle can be solved.

4-9. The Solution of Oblique Triangles. There are four combinations of angles and sides that will supply you with the information necessary for solving an oblique triangle. They are as follows:

- One side and two angles.

- Two sides and an opposite angle.

- Two sides and the included angle.

- Three sides.

a. Any triangle can be solved by dividing the triangle into two right triangles and applying the formulas for solving right triangles. To eliminate much of the work involved in solving two right triangles, formulas or laws have been derived that enable you to solve oblique triangles directly.

b. The sine law states that in any triangle, the sides are proportional to the sines of their opposite angles. This law also applies in solving triangles where the given information includes one side and two angles or two sides and an opposite angle. When applying the sine law to Figure 4-17, you get various relationships and equations. For example:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

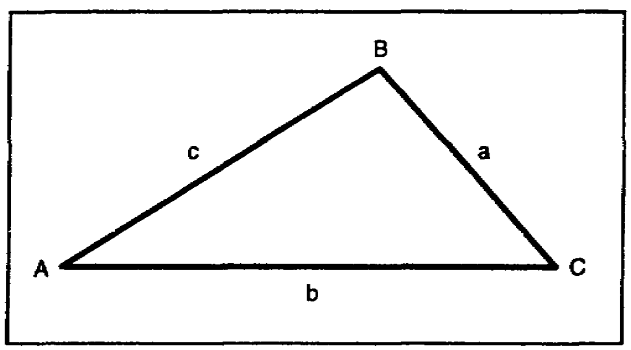


Figure 4-17. The oblique triangle

c. Since you can solve any triangle in which three parts are known, including a side, it is necessary to set up a relationship between any two sides and their opposite angles. These relationships are as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

d. The cosine law states that in any triangle, the square of any side is equal to the sum of the squares of the other sides minus twice the product of those two sides time the cosine of the angle included between them. This law is adaptable to solving triangles where two sides and the included angle are given and it is desired to find the length of the third side. Three situations exist where the cosine law can be applied. They are as follows:

$a^2 = b^2 + c^2 - 2\ bc \cos A$
$b^2 = a^2 + c^2 - 2\ ac \cos B$
$c^2 = b^2 + a^2 - 2\ ab \cos C$

If the given included angle is obtuse, remember that its cosine is negative and this sign must be applied when the value of the cosine is substituted in the formula.

e. The tangent law states that the sum of any two sides of a triangle is to their difference as the tangent of half the sum of their opposite angles is to the tangent of half the difference between their opposite angles. This law is best suited for solving triangles in which two sides and the included angle are given. The tangent law in equation form is as follows:

$$\frac{b+a}{b-a} = \frac{\tan \frac{1}{2}(B+A)}{\tan \frac{1}{2}(B-A)} \qquad \text{- OR -} \qquad \tan \tfrac{1}{2}(B-A) = \frac{b-a}{b+a}\left[\tan \tfrac{1}{2}(B+A)\right]$$

Since it is customary to avoid negative quantities when using the tangent law, you can rearrange the equation to eliminate this. If in the equation above side a had been longer than side b, the equation would have been as follows:

$$\tan \tfrac{1}{2}(A-B) = \frac{a-b}{a+b}\left[\tan \tfrac{1}{2}(A+B)\right]$$

f. The half-tangent formulas are used to solve the angles of a triangle in cases where the given information regarding the triangle is three sides. These formulas are as follows:

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan \tfrac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$$

In the formulas above, s is equal to one-half the summation of the three sides (a, b, and c).

g. In the following paragraphs, there are some typical cases in which the above laws will help you in determining the unknown quantities in oblique triangles.

4-10. Case I - One Side and Two Angles. To solve a triangle where the given information includes a side and two angles, find the value of the third angle by subtracting the sum of the two given angles from 180°. (The sum of the interior angles of a triangle equals 180°.) Then, using the sine law, solve the two unknown sides.

Example: In Figure 4-18, side a of the triangle equals 175, angle A equals 76° 30' 00", and angle B equals 48° 45' 00". Find angle C and sides b and c.

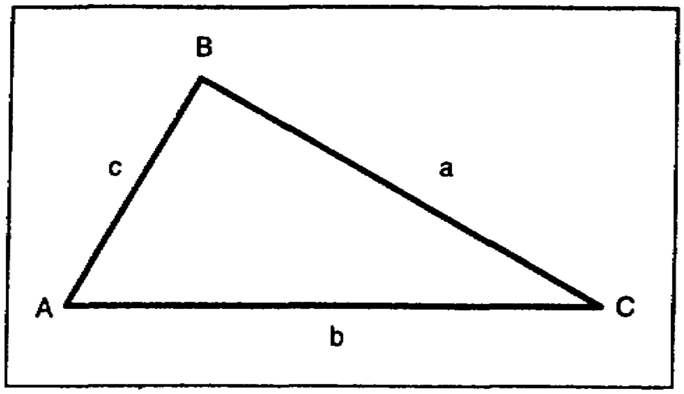


Figure 4-18. Case I

Solution: Find the value of angle C by subtracting the sum of angles A and B from 180°.

180° - (76° 30' 00" + 48° 45' 00") = 54° 45' 00" = C

Then, using the sine law, solve for the two unknown sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b = \frac{a \sin B}{\sin A} = \frac{(175)(\sin 48° 45' 00")}{\sin 76° 30' 00"}$$

$$= \frac{(175)(0.75183989)}{0.9723699}$$

$$= 135.3 \text{ (by natural functions)}$$

Also find side c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} = \frac{(175)(\sin 54° 45' 00")}{\sin 76° 30' 00"}$$

$$= \frac{(175)(0.8166416)}{0.9723699}$$

$$= 147.0 \text{ (by natural functions)}$$

Note that the same results are obtained when you use natural functions or logarithms of circular functions.

4-11. Case II - Two Sides and an Opposite Angle. In solving a triangle where two sides and an opposite angle are known, it is necessary to solve for one of the unknown angles using the sine law. Next, you will solve for the third angle by subtracting the sum of the given angle and the solved-for angle from 180°. Then, by using the sine law, you can solve for the remaining unknown side.

Example: In Figure 4-19, side b of the triangle equals 135.2 feet, side a equals 196.6 feet, and angle A equals 32° 36' 40". Find angles B and C and side c.

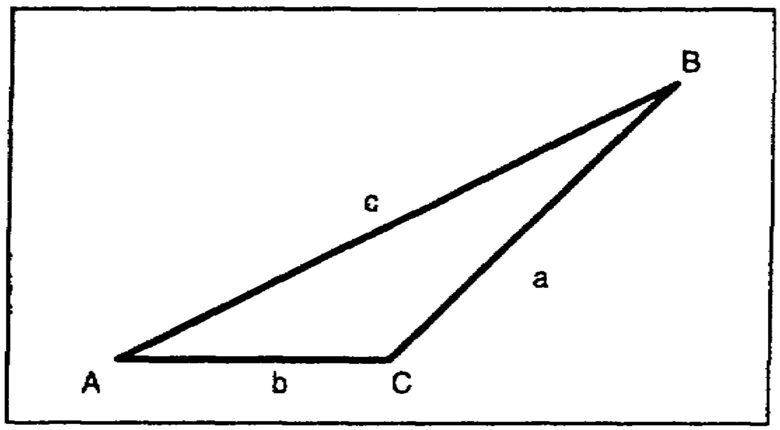


Figure 4-19. Case II

Solution: Using the sine law, solve for angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{196.6}{\sin 32^\circ\ 36' 40''} = \frac{135.2}{\sin B}$$

$$\sin B = \frac{(135.2)(\sin 32^\circ\ 36' 40'')}{196.6}$$

$$= \frac{(135.2)(0.539756)}{196.6}$$

$$= 0.3711185$$

B = 21° 45' 14" (by natural functions)

Then solve for angle C by subtracting the sum of angles A and B from 180°.

C = 180°-(32° 36' 40"+21° 45' 14") = 125° 38' 06"

Then solve for side c using the sine law.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A} = \frac{(196.6)(\sin 125° 38' 06")}{\sin 32° 36' 40"}$$

$$= \frac{(196.6)(0.8127450)}{0.539756}$$

c = 296.03 feet (by natural functions)

a. The sine of an acute angle is the same as the sine of the supplementary obtuse angle. Therefore, in solving a triangle under case II by the sine law, two values for the angle are possible and either value can be taken unless excluded by other conditions in the triangle.

b. In any triangle, only one of the angles can be obtuse. When the given angle is obtuse, both of the other angles are acute. In Figure 4-20, angle A of the triangle is acute, and side c times the sine of A equals the line BP. This is the side opposite the given angle in the right triangle ABP. It is also the altitude of the triangle in question; therefore, BC or BC^1 cannot be shorter than the altitude. When side a is less than side c times the sine of A, the triangle is impossible. When angle A is obtuse, side a must be longer than side c or the triangle is impossible. When the angle A is acute and the length of the triangle is shorter than side c but longer than side c times the sine of A (BP), the triangle is ambiguous and two solutions are possible.

4-12. Case III - Two Sides and the Included Angle. When solving for the third side of a triangle and two sides and the included angle are known, it is best to use the cosine law.

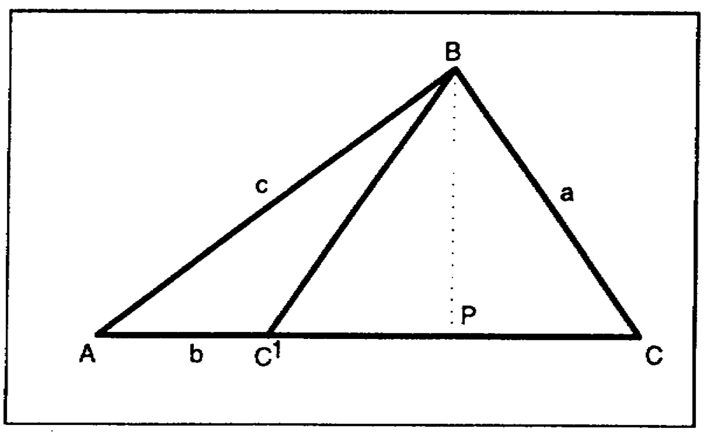


Figure 4-20. The ambiguous triangle

Example: In Figure 4-21, side b of the triangle equals 97.85 feet, side c equals 106.66 feet, and angle A equals 73° 19' 27". Find side a.

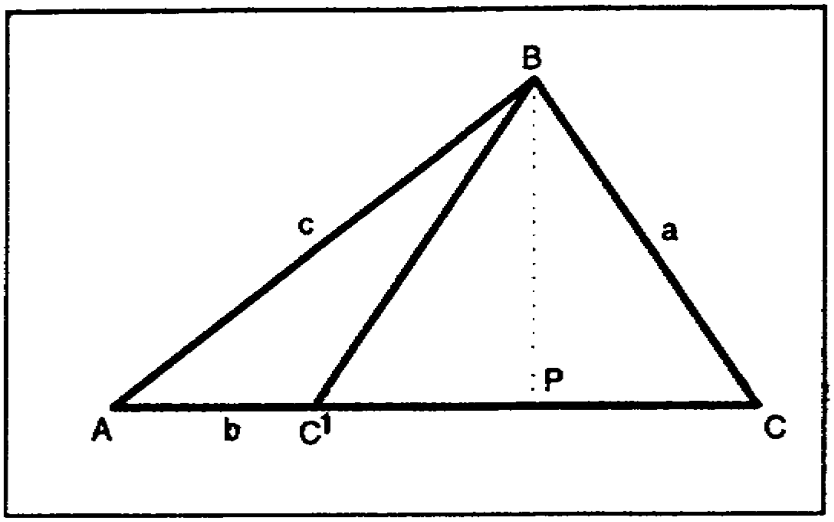


Figure 4-21. Case III

Solution:

$$a^2 = b^2 + c^2 - 2\ bc \cos A$$

$$a^2 = (97.85)^2 + (106.66)^2 - (2)(97.85)(106.66)(\cos\ 73°19'27")$$

$$a^2 = 9574.62 + 11376.36 - 5989.63 = 14961.35$$

$$a = \sqrt{14961.23} = 122.32 \text{ feet (by natural functions)}$$

Now combine the different parts of the equation.

$a^2 = b^2 + c^2$ -2 be cos A
a^2 = (9574.62 + 11376.36) -5989.63 = 14961.35

When solving for the remaining two unknown angles of the triangle and two sides and the included angle are known, it is best to use the tangent law in the solution.

Example: In Figure 4-22, page 30, side c of the triangle equals 749.63 feet, side a equals 561.88 feet, and angle B equals 41° 17' 32". Find angle A and angle C.

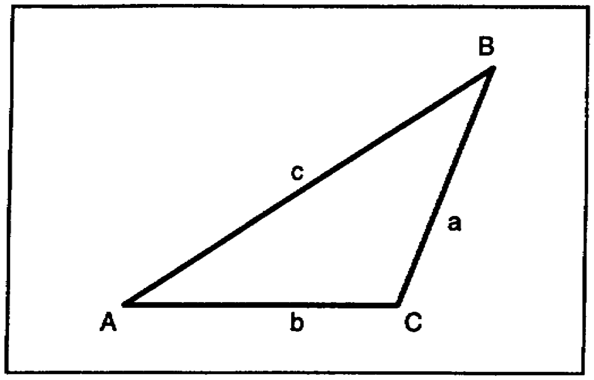


Figure 4-22. Solving for unknown angles (tangent law)

Solution: Find the value of angles A and C by subtracting angle B from 180°.

(C = A) = 180°-41° 17' 32"= 138° 42' 28"

Then substitute the known values in the tangent equation.

$$\frac{c+a}{c-a} = \frac{\tan \frac{1}{2}(C+A)}{\tan \frac{1}{2}(C-A)}$$

$$\tan \tfrac{1}{2}(C-A) = \frac{c-a}{c+a}\left[\tan \tfrac{1}{2}(C+A)\right]$$

$$= \frac{749.63-561.88}{749.63+561.88}(\tan 69^\circ 21' 14'')$$

$$\tan \tfrac{1}{2}(C-A) = 0.379899$$

$$\tfrac{1}{2}(C-A) = 20^\circ 48' 12''$$

Now find the values of the unknown angles from the equations which follow:

$$C = \tfrac{1}{2}(C+A) + \tfrac{1}{2}(C-A)$$

$$= 69^\circ 21'\ 14'' + 20^\circ 48'\ 12'' = 90^\circ 09'\ 26''$$

$$A = \tfrac{1}{2}(C+A) - \tfrac{1}{2}(C-A)$$

$$= 69^\circ 21'\ 14'' - 20^\circ 48'\ 12'' = 48^\circ 33'\ 02''$$

The above solution is by natural functions.

4-13. Case IV - Three Sides. When three sides of a triangle are known, it is best to solve for the remaining parts by the method of half tangents.

Example: In Figure 4-23, side a of the triangle equals 197.70, side b equals 206.15, and side c equals 184.42. Find the value of the three angles.

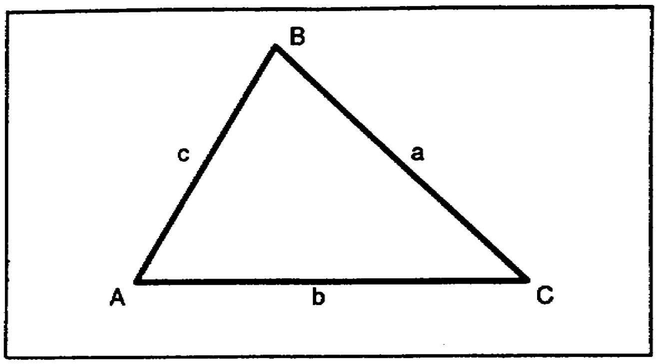


Figure 4-23. Case IV

Solution: Determine the value of s.

$$s = \frac{a + b + c}{2} = \frac{197.70 + 206.15 + 184.42}{2} = 294.135$$

Now solve for the angles.

$$\tan \tfrac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= \sqrt{\frac{(294.135 - 197.70)(294.135 - 184.42)}{(294.135)(294.135 - 206.15)}}$$

$$= \sqrt{\frac{(96.435)(109.715)}{(294.135)(87.985)}} = \sqrt{\frac{10580.366}{25879.468}}$$

$$= \sqrt{0.4088324} = 0.6394000$$

$$\tfrac{1}{2} B = 32° \, 35' \, 41.5''$$

$$B = 65° \, 11' \, 23''$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= \sqrt{\frac{(294.135 - 206.15)(294.135 - 184.42)}{(294.135)(294.135 - 197.70)}}$$

$$= \sqrt{\frac{(87.985)(109.715)}{(294.135)(96.435)}} = \sqrt{\frac{9653.274}{28364.909}}$$

$$= \sqrt{0.3403225} = 0.5833734$$

$$\tfrac{1}{2} A = 30° \ 15' \ 29.5''$$

$$A = 60° \ 30' \ 59''$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{(s - a)(s - b)}{s(s - c)}}$$

$$= \sqrt{\frac{(294.135 - 197.70)(294.135 - 206.15)}{(294.135)(294.135 - 184.42)}}$$

$$= \sqrt{\frac{(96.435)(87.985)}{(294.135)(109.715)}} = \sqrt{\frac{8484.833}{32271.022}}$$

$$= \sqrt{0.2629242} = 0.5127613$$

$$\tfrac{1}{2} C = 27° \ 08' \ 49''$$

$$C = 54° \ 17' \ 38''$$

Check your solution by adding angles A, B, and C to see if they equal 180°.

(60° 30' 59") +(65° 11' 23") +(54° 17' 38") = 180°

The third angle could also be fund by subtracting the first two angles from 180°. This, however, leaves no way to check the accuracy of the work performed.

LESSON 4

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. There is only one correct answer to each item. When you complete the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. Which of the following would be the result of subtracting 98° 47' 52" from 36° 40' 30"?

   A. 62° 07' 22"
   B. 135° 28' 43"
   C. 197° 35' 44"
   D. 297° 52' 38"

2. If you divide an angle of 35° 14' 15" into three equal segments, how many degrees, minutes, and seconds would each segment have?

   A. 11° 18' 45"
   B. 11° 44' 45"
   C. 11° 52' 45"
   D. 12° 00' 45"

3. If A and B are the acute angles of a right triangle, then cosine A is equal to _______.

   A. cos B
   B. sin B
   C. sec B
   D. sec A

4. In which of the four quadrants of a circle would an angle containing 282° 16' be located?

   A. IV
   B. II
   C. III
   D. I

5. To solve a right triangle, two parts besides the right angle must be known. One of these parts must be __________.

   A. An obtuse angle
   B. The hypotenuse
   C. An exterior angle
   D. A side

LESSON 4

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct answer and feedback	
1.	D	297° 52' 38" Accurately subtracting two ... (page 4-6, para 4-3e)
2.	B	11° 44' 45" The original problem ... (page 4-2, para 4-2a)
3.	B	sin B To differentiate between ... (page 4-8, para 4-4d)
4.	A	IV Referring to Figure 4-11 ... (page 4-16, para 4-6b)
5.	D	A side As you will recall ... (page 4-13, para 4-5h)

LESSON 5

SURVEYING EQUIPMENT

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will become familiar with surveying equipment.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will become familiar with the different types of equipment used in surveying operations.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Accuracy in surveying is essential because other engineering factors that are involved after the survey is complete depend on the surveying results. Construction surveys deal with determining the relative positions of points on the earth's surface. These points are used to locate and lay out roads, airfields, and man-made structures (such as buildings, sewer lines, utility lines, and any type of proposed or existing structure). Construction surveys also identify terrain features that are used to draw large-scale maps. No matter what type of survey is needed, some type of surveying equipment will be required. Each surveying operation requires certain specific types of equipment. These surveys can vary in many ways, depending on the technical requirements needed. This lesson will familiarize you with the surveying equipment that you will encounter in the field.

PART A - UNIVERSAL SURVEYING INSTRUMENTS

5-1. Tools and Devices. Primarily, the surveying instruments that you will be using are precise tools with which measurements are made. Many of the instruments have similar features. These instruments include

- Tripods to hold the instrument steady at a convenient height.

- Level vials and leveling thumb screws to make the telescope level and parallel to a horizontal line of sight.

- Optical devices, such as crosshairs in a telescope to sight on a target.

- Magnification lenses on the telescope to magnify a target.

- Plumb bobs or optical plummets to align the instrument exactly over a selected point.

The surveying instruments that you will become familiar with are the one-minute theodolite, geodimeter (total station), and levels. These items of equipment are needed to help you determine the angles and elevations that are required for a construction survey.

a. One-Minute Theodolite. The one-minute theodolite is used to obtain both horizontal and vertical angles. The theodolite is a compact, lightweight, dustproof optical-reading, directional-type instrument (see Figure 5-1). It may also be used as a repeating-type instrument for measuring horizontal angles by using the repeating damp. The scales are readable directly to the nearest minute and may be illuminated by sunlight or artificial light. The surveying points are observed through the instrument and the angles read through an optical microscope on horizontal and vertical scales inside the instrument. There are two versions of this theodolite-the engineer version, which reads directly to one minute (more commonly known as the T16) and the artillery version, which reads directly to 0.2 mil (more commonly known as the T2).

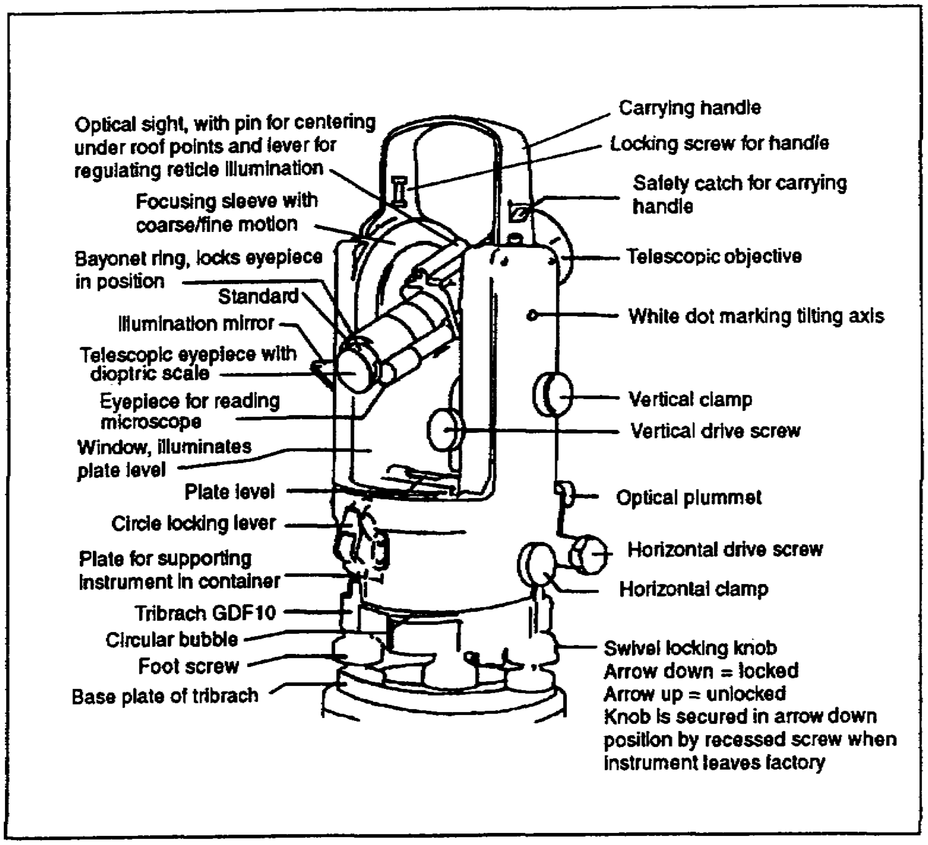


Figure 5-1. One-minute theodolite

b. Geodimeter. The geodimeter is an automated, integrated surveying instrument (AISI) commonly referred to as a total station (see Figure 5-2, page 5-4). It combines a theodolite with an electronic distance-measurement (EDM) instrument and an electronic data collector. The geodimeter can measure angles up to one-second accuracy, distances (up to 3 kilometers) to the thousandths, and elevations to the hundredths of a foot or meter. It has the capacity to store up to 6,000 points of surveying information. It can operate up to eight hours continuously on the batteries provided, or a 12-volt car battery can be used. The geodimeter comes with factory-set programs designed for standard surveying operations and can be programmed for special surveying operations. The geodimeter is used in conjunction with the civil software program Terramodel. This software accepts the downloaded information from the geodimeter and allows rapid development of the surveying site by producing civil engineering drawings.

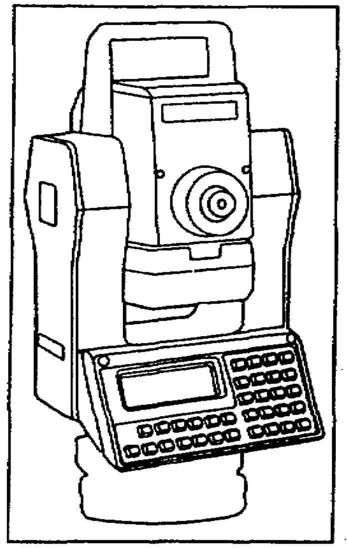


Figure 5-2. The geodimeter (AISI)

c. Levels. Discussed in the following paragraphs are the different types of levels that may be used in construction surveys.

(1) Dumpy Level. The dumpy level is a very sturdy and reliable instrument that was used extensively for leveling operations until it was replaced by more modern equipment (see Figure 5-3). Its sighting device is a 28 variable-power telescope with a maximum length of 18 inches and an erecting eyepiece that changes the inverted/upside-down image so that it can be seen right-side up. The focusing knob is normally external. Rotating the focusing knob brings the target into clear focus. The reticle has two crosshairs at right angles to each other, and some models have stadia hairs for distance measurement to the nearest foot or meter. Rotating the eyepiece brings the crosshairs into focus. The telescope and level-bar assembly are mounted on a spindle that permits the unit to be moved only in a horizontal plane. The telescope and level bar cannot be elevated or depressed. The dumpy level's telescope is rigidly attached to the level bar that holds an adjustable, highly sensitive level vial. The azimuth clamp and azimuth tangent screw allow for slow motion of the telescope for accurate centering on a target. The spindle mounts in a four-screw leveling head that rests on a footplate. The footplate screws onto the threads of the tripod. When the instrument is properly leveled and adjusted, the horizontal line of site, which is defined by the horizontal crosshair, forms a horizontal plane.

(2) Automatic Level. The automatic level (also called the auto level) is a self leveling level that has become the most popular, standard-type level used in construction surveys (see Figure 5-4). It is very easy to use and can be set up quickly. The automatic level has upgraded surveying operations by taking the place of the dumpy level, which requires a great amount of time to center its bubble and reset its position. The automatic level has a small circular level called a bull's-eye level and only three leveling screws. The leveling screws are on a triangular footplate and are used to center the bubble in the bulls-eye level. The line of sight automatically becomes horizontal and remains horizontal as long as the bubble stays centered. Inside the automatic level, a gravity-suspended prism (called a compensator) is hung on fine, nonmagnetic wires. The action of gravity on the compensator causes the

prism to swing freely into a position so that a horizontal line of sight is achieved. This horizontal line of sight is maintained even when the telescope is unlevel or the instrument is disturbed because the prism swings freely with gravity.

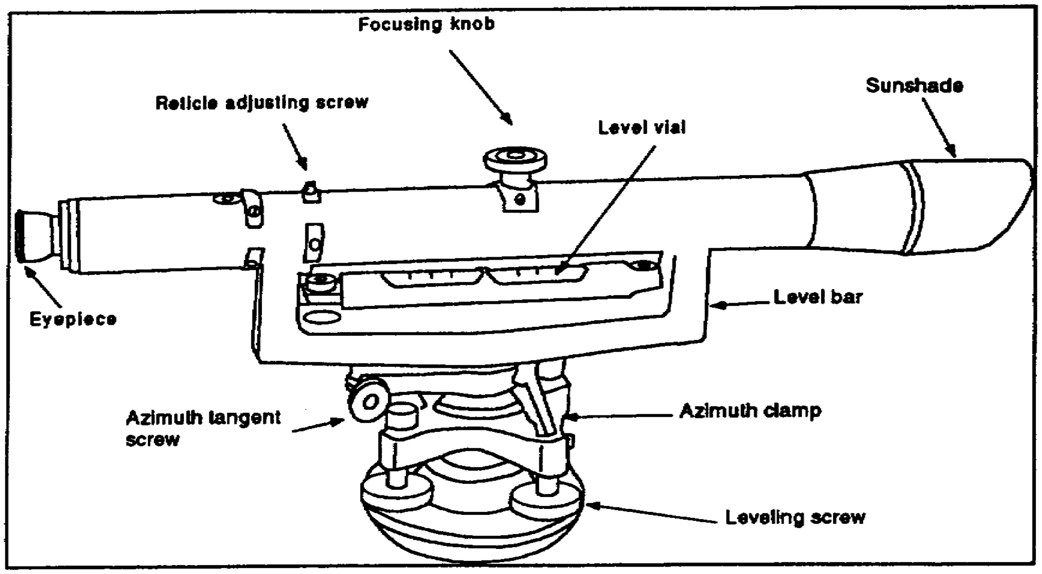


Figure 5-3. The dumpy level

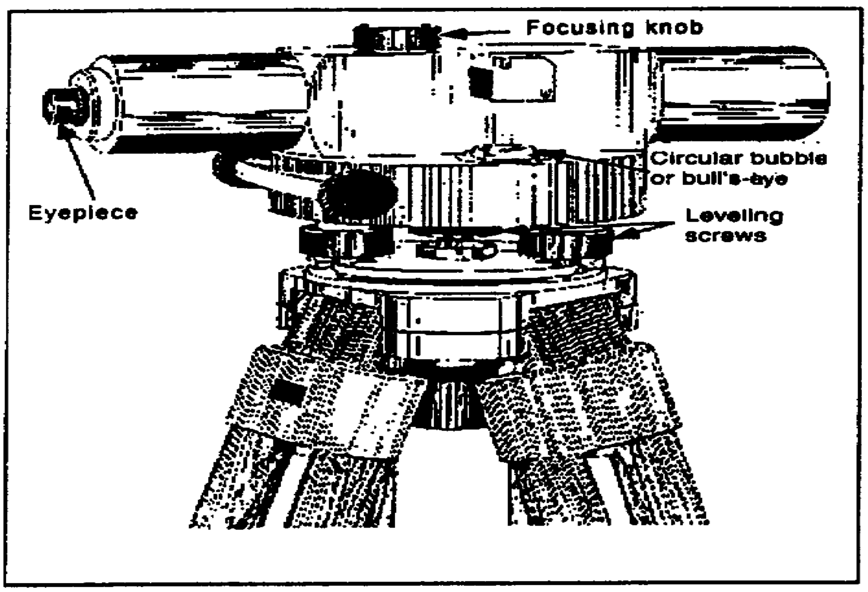


Figure 5-4. The automatic level

(3) Hand Levels. The hand level, like all surveying levels, is an instrument that combines a level vial and a sighting device (see Figure 5-5, page 5-6). It is a very simplistic device that is used for taping and for rough determinations of elevations. The basic principle is that if the bubble is centered while sighting through the tube, the line of sight is horizontal.

(a) For example, in cross-sectional work, terrain irregularities may cause elevations to go beyond the automatic level's range from a setup. A hand level is useful for extending approximate elevations off the control-survey line beyond the limits of the automatic level. For greater stability, the hand level may be rested against a tree, a Philadelphia rod, or a range pole.

(b) On a hand level, a level vial is mounted atop a slot in the sight tube in which a reflector is set on a 45° mirror. This allows the observer, while sighting through the tube, to see the landscape or object, the position of the bubble in the vial, and the index line at the same time. The hand level has no magnification capabilities, therefore, the distances sighted are relatively short.

(c) In Figure 5-5, view A shows a Locke hand level that is very simplistic in design and view B shows an Abney hand level (or clinometer) which has a reversible, graduated arc assembly mounted on one side and may be used for measuring vertical angles and percent of slope. The lower side of the arc is graduated in degrees, and the upper side is graduated in percent of slope. The level vial of the Abney hand level is attached at the axis of rotation of the index arm. When the index arm is set to zero, the instrument is used like a Locke hand level. When it is used as a clinometer, the object is sighted and the level tube is rotated about the axis of rotation until the bubble is centered. The difference between the line of sight and the level-bubble axis can be read in degrees or percent of slope from the position of the index arm.

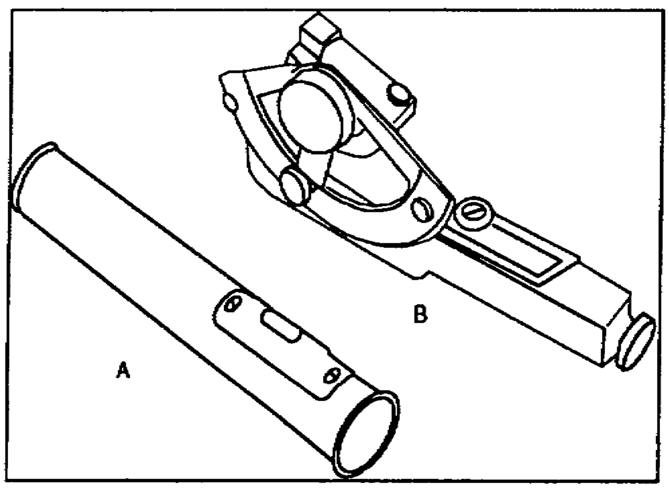


Figure 5-5. Hand levels

PART B - FIELD EQUIPMENT

5-2. Hand Tools. The term field equipment, as used in this lesson, includes all devices, tools, and instrument accessories used in connection with field measurements.

a. When conducting a survey across rough terrain, various types of tools will be needed to clear the line, that is, cut down brush and other natural growth as needed (see Figure 5-6). Surveying procedures usually permit the bypassing of large trees. However, it may be necessary to fell a tree. If heavy equipment is working in the area, it may be used to fell the tree; if not, a chain saw may be used. If a chain saw is not available, use an ax.

The machete and brush hook may be used to clear small saplings, bushes, or similar growth. Axes and hatchets are used to mark trees by blazing and may also be used to fell trees.

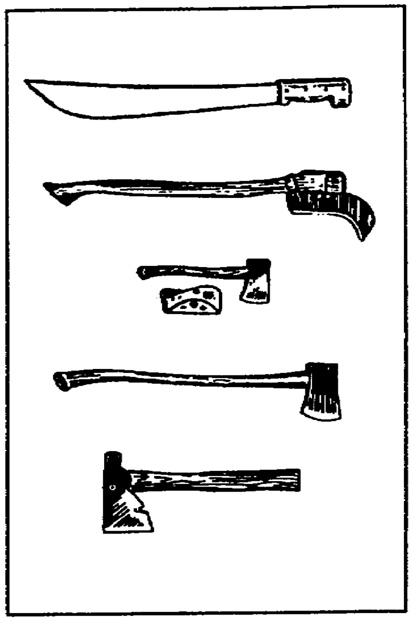


Figure 5-6. Cutting tools

b. A hatchet or single-bit ax is used to drive hubs, stakes, pipe, and other markers into the ground. A sledgehammer, however, is a more suitable tool for this purpose. A double-faced, long-handled sledgehammer is shown in Figure 5-7, page 5-8. It is swung with both hands. There are also short-handled sledgehammers that can be swung with one hand. A sledgehammer is classified according to the weight of the head. Common weights are 6, 8, 10, 12, 14, and 16 pounds. The 8- and 10- pound weights are the most commonly used.

c. When searching for hidden markers, you may need a shovel or a pick like the ones shown in Figure 5-7, page 5-8, to clear off the topsoil. In soft ground, such as loose, sandy soil, you may prefer to use a square-headed shovel or a probing steel rod to locate buried markers.

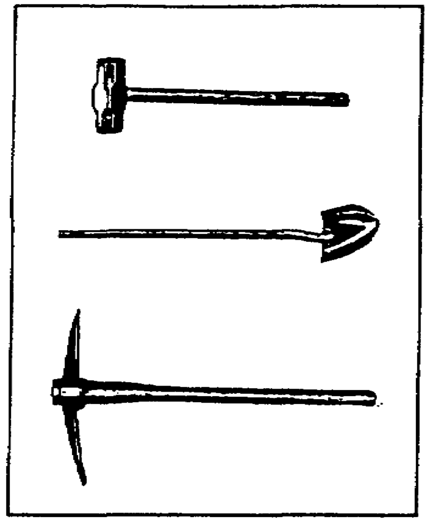


Figure 5-7. Sledgehammer, shovel, and pick

PART C - ASSOCIATED SURVEYING EQUIPMENT

5-3. Surveying Tapes. Tapes are used in surveying to measure horizontal, vertical, and slope distances. They may be made of a ribbon or band of steel, an alloy of steel, cloth reinforced with metal, or synthetic materials. Tapes are issued in various lengths and widths and graduated in a variety of ways. Various types of surveying tapes are shown in Figure 5-8. View A shows a nonmetallic tape, view B shows a steel tape on an open reel, view C shows a metallic tape on a closed reel, and View D shows a special type of low-expansion steel tape (called an Invar tape or a Lovar tape) used for geodetic work and for checking the length of regular steel tapes. The Invar and Lovar tapes are very precise and will not react to temperature changes.

a. Metallic Tape. A metallic tape is made of high-grade synthetic material with strong wire strands (bronze, brass, and copper) woven in the warped face of the tape and coated with a tough, water-resistant plastic for durability. Standard lengths are 50 and 100 feet. Most metallic tapes are graduated in feet and decimals of feet, but some are graduated in feet and inches to the nearest 1/4 inch, meters, and centimeters. Metallic tapes are generally used for rough measurements, such as cross-sectional work, roadwork slope staking, and side shots in topographic surveys.

b. Nonmetallic Tapes. Some surveyors prefer to use nonmetallic tapes that are woven from synthetic yarn, such as nylon, and coated with plastic. Nonmetallic tapes are similar to metallic tapes in their use, lengths, and graduations.

c. Steel Tapes. For direct linear measurements of ordinary or more accurate precision, a steel tape is required. The most commonly used length is 100 feet, but tapes are also available in 50-, 200-, 300-, and 500-foot lengths. All tapes except the 500-foot tape are band-type; the common band widths are 1/4 and 5/16 inch. The 500-foot tape is usually a flat-wire type. Metric steel tapes are available and are commonly used overseas, with the most common lengths being 30 and 50 meters. Most steel tapes are graduated in feet and decimals of feet, but some are graduated in feet and inches, meters, or other linear units. Steel tapes are sometimes equipped with a reel on which they can be wound. These tapes can be, and often are, detached from the reel for more convenient use in taping.

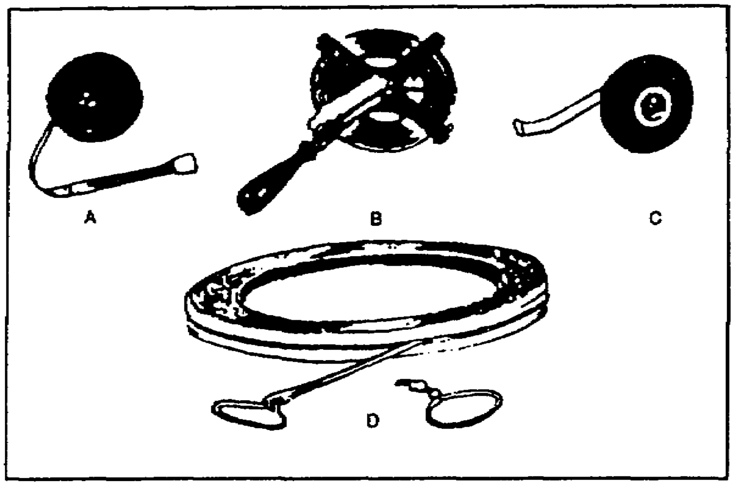


Figure 5-8. Surveying tapes

5-4. Surveying Accessories. Surveying accessories include the equipment, tools, and other devices used in surveying that are not considered to be an integral part of the surveying instrument itself. For example, when you run a traverse, your primary instruments may be the theodolite and the steel tape. The accessories you will need to do the actual measurement are-

- A tripod to support the theodolite.
- A range pole to sight on.
- A plumb bob to center the instrument on the point.
- Tape supports to support the tape from sagging if the survey is of high precision.

It is important that you become familiar with the proper care of this equipment and use it properly.

a. Tripod. The tripod is the base or foundation that supports the surveying instrument and keeps it stable during observations. A tripod consists of a head to which the instrument is attached, three wooden or metal legs that are hinged at the head, and pointed metal shoes on each leg to be pressed or anchored into the ground to achieve a firm setup (see Figure 5-9, page 5-10). The leg hinge is adjusted so that the leg will just begin to fall slowly when it is raised to an angle of about 45°. The tripod head may have screw threads on which the instrument is mounted directly, a screw protecting upward through the plate, or a hole or slot through which a special bolt is inserted to attach to the instrument. When mounting the instrument on the tripod, firmly grip it to avoid dropping it. Hold the theodolite by the right standard (opposite the vertical circle) while you are attaching it. The automatic and dumpy levels should be held at the center of the telescope. Both theodolites and levels should be gripped near the base of the instrument with the opposite hand. The instrument should be screwed down to a firm bearing but not so tightly that it will bind or the screw threads will strip.

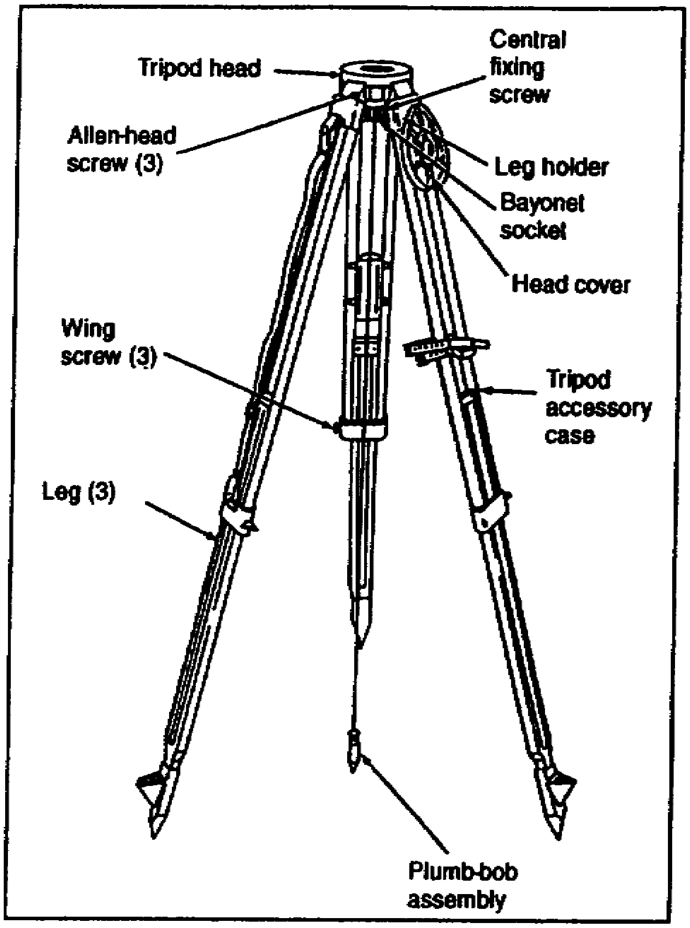


Figure 5-9. Tripod detail

(1) Types. There are two types of tripods that surveyors use: the fixed leg and the extension leg (see Figure 5-10). These tripods can also have wide frames like those shown in Figure 5-11, which have greater torsional stability and tend to vibrate less in the wind.

(a) Fixed Leg. The fixed-leg tripod is also called a stilt-leg or rigid tripod. Each fixed leg may consist of two lengths of wood as a unit or a single length of wood split at the top, which is attached to a hinged tripod head fitting and to a metal shoe. At points along the length of the leg, perpendicular brace pieces are sometimes added to give greater stability. The fixed legs must be swung in or out in varying amounts to level the head. Instrument height is not easily controlled, and the observer must learn the correct spread of the legs to get the desired height.

(b) Extension Leg. The extension-leg tripod is also called a jack-leg tripod. Each extension leg is made of two sections that slide longitudinally. On rough ground, the legs are adjusted to different lengths to establish a horizontal tripod head or to set the instrument at the most comfortable working height for the observer. A leg may be shortened and set as shown in the extreme left view of Figure 5-10.

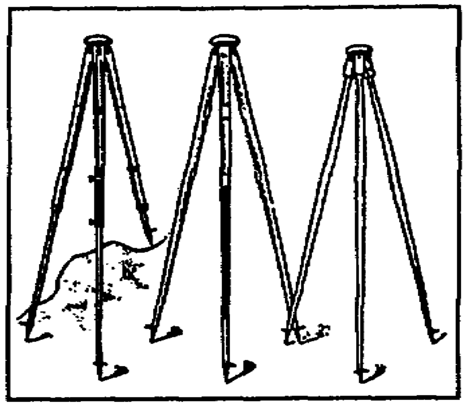


Figure 5-10. Fixed- and extension-leg tripods

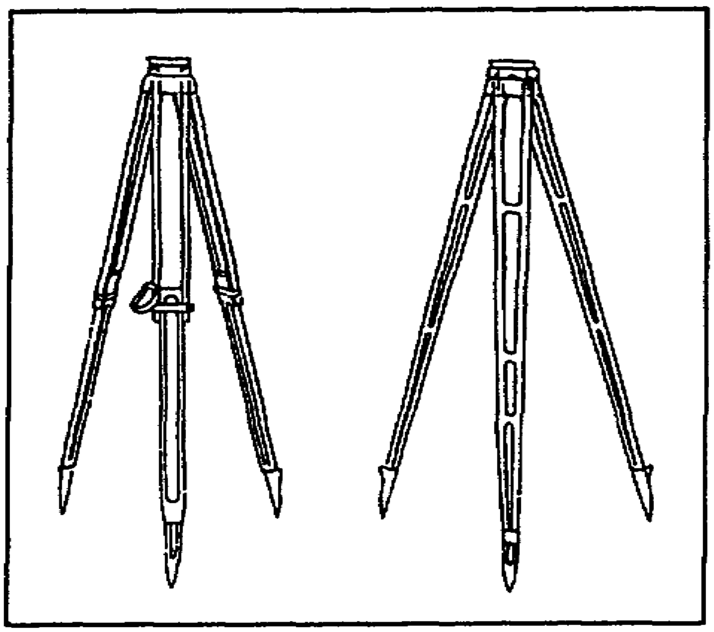


Figure 5-11. Wide-frame tripods

(2) Setup. When setting up a tripod, you should be sure to place the legs so that you achieve a stable setup. First, loosen the restraining strap from around the three legs and secure it around one leg. While standing over the setup mark, grip the tripod with two of the legs close to the body and, by using one hand, push the third leg out away from the body until it is about 3 feet from the mark. Lower the tripod until the third leg is on the ground. Place one hand on each of the first two legs and spread them while taking a short backward step, using the third leg as a pivot point. When the two legs look about as far away from the mark as the third one and all three are about equally spaced, lower the two legs and press them into the ground. Make any slight adjustment to level the head further by moving the third leg a few inches in or out before pressing it into the ground.

(a) On smooth or slippery surfaces, you should tighten the tripod's leg hinges while setting it up to prevent the legs from spreading and causing the tripod to fall. If there are holes or cracks in the ground, use them to brace the tripod. Sometimes, as a safety factor, you should tie the three legs together or brace them with rocks or bushes after they are set to keep them from spreading. If setups are to be made on a slippery finished floor, rubber shoes may be fitted to the metal shoes or an equilateral triangle leg retainer may be used to prevent the legs from sliding.

(b) When you are setting up on sloping ground, place the third leg uphill and at a greater distance from the mark. Set the other two legs as before, but before releasing them, make sure that the weight of the instrument and the tripod head does not overbalance the tripod and cause it to slip or fall.

(c) Proper care must be observed in handling the tripod. When the legs are set in the ground, apply pressure longitudinally. Pressure across the leg can crack the wooden pieces. The hinge joint should be adjusted and not over tightened to the degree that it would cause strain on the joint or strip or lock the metal threads. The tripod head should be kept covered with the head cover or protective cap when not in use, and the head should not be scratched or burred by mishandling. When the tripod is in use, the protective cap is to be placed in the instrument box to prevent it from being misplaced or damaged. Any damage to the protective cap can be transferred to the tripod head. Mud, clay, or sand adhering to the tripod must be removed, and the tripod should be wiped with a damp cloth and dried. The metal parts should be coated with a light film of oil or wiped with an oily cloth. Foreign matter can get into hinged joints or on smooth surfaces and cause wear. Stability is the tripod's greatest asset. Instability, wear, or damaged bearing surfaces on the tripod can evolve into unexplainable errors in the final surveying results.

b. Range Poles. A range pole is a wood, fiberglass, or metal pole, usually about 8 feet long and about 1/2 to 1 inch in diameter. It has a steel-weighted point and is painted in alternate bands of red and white to increase its visibility. The bands are 1 foot long and can be used as a rough measurement guide using stadia estimation. Figure 5-12 shows a variety of range poles. The range pole is held vertically on a point or plumbed over a point so that the point may be observed through an optical instrument It is primarily used as a sighting rod for either linear or angular measurements. For work of ordinary precision, chainmen may stay on line by observing a range pole.

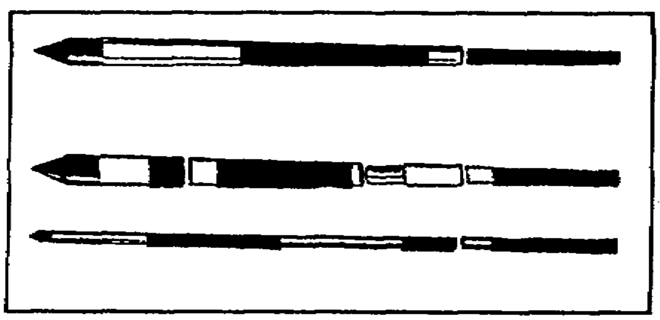


Figure 5-12. Range poles

c. Plumb Bob, Cord, and Target. A plumb bob is a pointed, tapered brass or bronze weight that is suspended from a cord to determine the plumb line from a point on the ground. Common weights for the plumb bobs are 6, 8, 10, 12, 14, 16, 18, and 24 ounce; the 12- and 16-ounce weights are the most popular. Typical plumb bobs are shown in Figure 5-13.

(1) A plumb bob is a precise instrument and must be cared for as such. If the tip becomes bent, the cord from which the bob is suspended will not occupy the true plumb line over the point indicated by the tip. A plumb bob usually has a detachable tip, a shown in Figure 5-13. If the tip becomes damaged, it can be renewed without replacing the entire instrument.

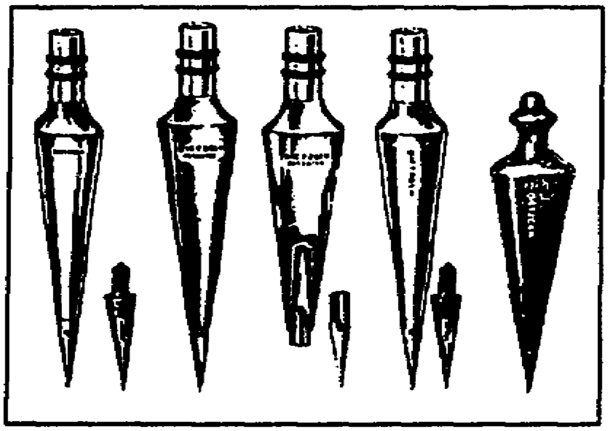


Figure 5-13. Plumb bobs

(2) Each survey party member should be equipped with a leather sheath for the plumb bob, and it should be placed in the sheath whenever it is not in use. The cord from a plumb bob can be made more conspicuous for observation purposes by attaching an oval-shaped aluminum target (Figure 5-14, view A, page 5-14). The oval-shaped target has reinforced edges with alternate red and white quadrants on is face. Also, a flat rectangular plastic target may be used (Figure 5-14, view B, page 5-14). It has rounded comers with alternate red and white quadrants on its face. These plumb-bob string targets are pocket size (approximately 2 by 4 inch).

d. Optical Plummet. The optical plummet is a device built into the theodolite or the tribrach of some instruments to center the instrument over a point. Its working principle is shown n Figure 5-15, page 5-14.

The plummet consists of a small prismatic telescope with a crosshair or marked-circle reticle adjusted to be in line with the vertical axis of the instrument. After the instrument is leveled, a sighting trough the plummet will check the centering over a point quickly. The advantages of the plummet over the plumb bob are that it permits the observer to center over a point from the height of the instrument stand and that it is not affected by the wind. A plumb bob requires someone at ground level to steady it and to inform the observer on the platform how to move the instrument and when it is exactly over the point. With the plummet, the centering and checking is done by the observer.

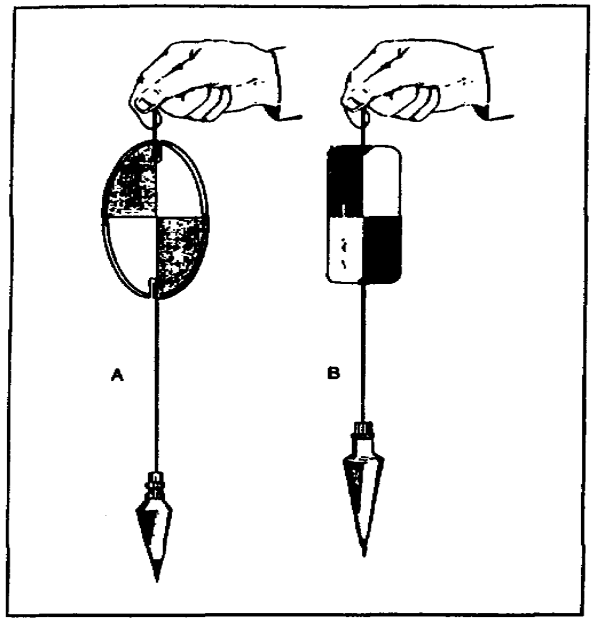


Figure 5-14. Plumb bob, cord, and target

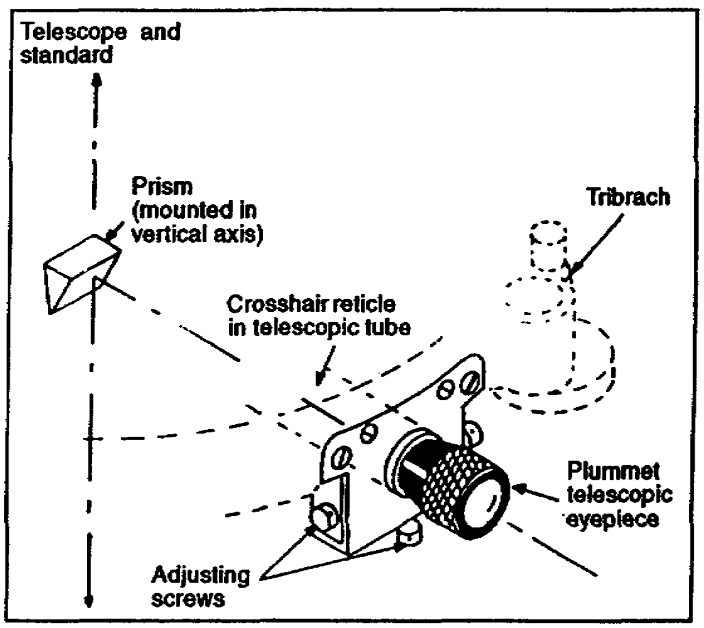


Figure 5-15. Optical plummet

e. Tape Clamp. There is usually a leather thong at each end of a tape that can be held when the full length of the tape is used. When only part of the tape is used, the zero end can be held by the thong, and the tape can be held at an intermediate point by means of a tape clamp (see Figure 5-16). These scissor-type clamps grip the tape tightly without bending or damaging it. The tape clamps are especially helpful when the tape needs to be pulled tightly. They make the tape easy to grip during measurements.

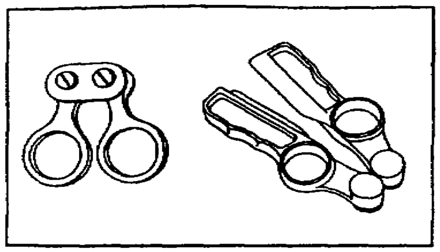


Figure 5-16. Tape clamps

f. Tension Scale. When a steel tape is held above ground between two crew members without support throughout, a certain amount of tension must be applied to reduce the sag in the tape. This can be done by using a tension scale, which is graduated in pounds from 0 to 30 (see Figure 5-17). It is clipped to the eye at the end of the tape, and tension is applied until the desired reading appears on the scale. The proper amount of continuous tension that needs to be applied to a steel tape is 20 pounds.

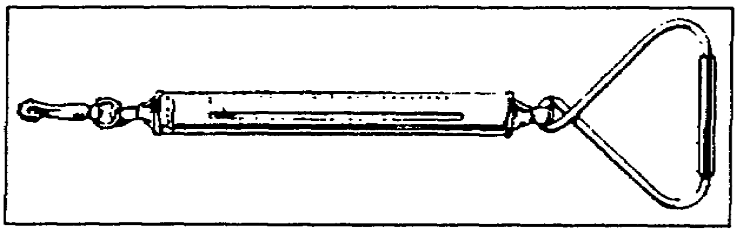


Figure 5-17. Tension scale

g. Taping Pin. A taping pin is a metal pin that is 1 foot long. It has a circular eye at one end and a point for pushing it into the ground at the other (see Figure 5-18, page 5-16). These pins come in sets of 11 and are carried on a wire ring that is passed through the eyes of the pins. Taping pins can be used to temporarily mark points in a great variety of situations. They are also used to keep count of tape increments in the taping of long distances. Each pin represents one tape length measured but not necessarily a full tape length. The number of taping pins used equals the number of distances that are measured and recorded in the recording book.

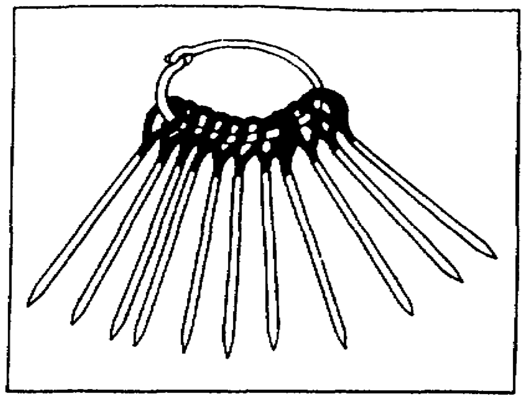


Figure 5-18. Taping pins

h. Leveling Rods. A leveling rod is a wooden rod that is used to measure the vertical distance above a point. This point may be a permanent elevation (bench mark), a temporary bench mark (a turning pin or stake), a man-made object, a constructed surface, or a natural point on the ground surface.

(1) Use. The leveling rod may be read directly by the instrument man sighting through the telescope, or it may be target read. Conditions that hinder direct reading, such as poor visibility, long sights, and partially obstructed sights (through brush or leaves), sometimes make it necessary to use a target. A target is also used to mark a rod reading when numerous points are set to the same elevation or a certain constant grade is needed from one instrument setup. In Figure 5-19, view B shows a rod with metric measurements; the graduations of the rod are in meters, decimeters, and centimeters. The targets that are furnished with the metric rod have a vernier that permits reading the scale to the nearest millimeter. The metric rod can be extended from 2.0 to 3.7 meters.

(2) Types. The different types of leveling rods are discussed in the following paragraphs.

(a) Philadelphia Rod. The most popular of all is the Philadelphia rod, which is a graduated two-section wooden rod (see Figure 5-19, view A). It can be extended from 7 to 13 feet and each foot is subdivided into hundredths of a foot. Instead of each hundredth being marked with a line or tick, the distance between alternate ones is painted black on a white background. Thus, the value for each hundredth is the distance between the colors; the top of the black increment is even values and the bottom of the black increment is odd values. The tenths are numbered in black and the feet are numbered in red. This rod may be used with the level, theodolite, and hand levels on occasion to measure the difference in elevation or it may be used for topographic land surveys.

Targets for the Philadelphia rod are usually oval, with the long axis at right angles to the rod and the quadrants of the target painted alternately red and white. The target is held in place on the rod by a C-clamp and a thumbscrew. A lever on the face of the target is used for fine adjustment of the target to the line of sight of the level. The targets have rectangular openings about the width of the rod and 0.15 feet high through which the face of the rod may be seen. A linear vernier scale is mounted on the edge of the opening with the zero on the horizontal line of the target for reading to thousandths of a foot. When the target is used, the rodman takes the rod reading.

(b) San Francisco Rod. The San Francisco rod is used for direct reading only and is available with three sliding sections.

(c) Chicago Rod. The Chicago rod is available with three or four sections that, instead of sliding, are joined at the end to each other like a fishing rod.

(d) Lenker Rod. The Lenker rod is a two-section rod similar to the Philadelphia but is graduated in feet and inches to the nearest one-eighth inch rather than the decimal. The upper section of the Lenker rod has the graduations on a continuous metal belt that can be rotated to set any desired graduation at the level of the height of the instrument (HI). To use the rod, set it on the bench mark and bring the graduation that indicates the elevation of the bench mark level with the HI. As long as the level remains at that same setup whenever you set the rod on a point, you can read the elevation of the point directly. In short, the Lenker rod does away with the necessity for computing the elevations.

(e) Lovar Rod. The Lovar rod is a high-precision leveling rod. It is usually T-shaped in cross section and has the scale inscribed on the metal strip. High-precision leveling rods usually have tapering, hardened-steel bases and some are equipped with thermometers so that the temperature correction can be applied. These rods generally contain built-in rod levels.

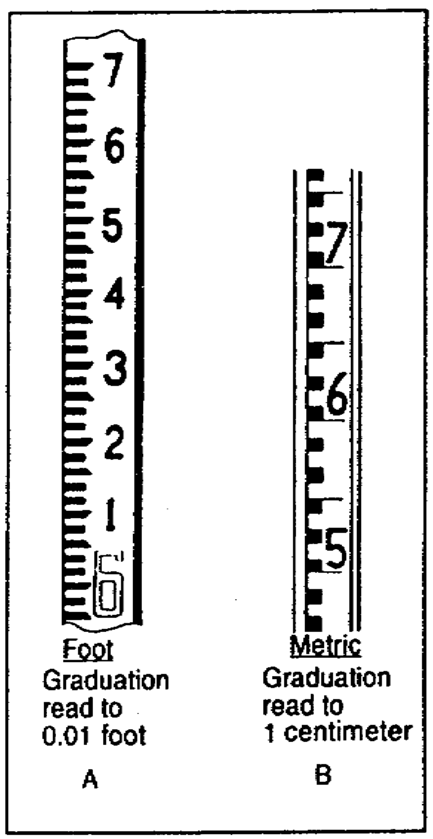


Figure 5-19. Leveling rods

(3) Care. Proper care should be taken of the leveling rods, such as keeping them clean (free of sand and dirt), straight, and readable. Leveling rods must be carried over the shoulder or under the arm from point to point. They must be collapsed in their original configuration when carrying them over long distances or when transporting them. Dragging them through the brush or along the ground will wear away or chip the paint. Do not set the rod with the numbers face down, as this will wear off the painted numbers. When not in use, the leveling rods should be stored in their cases, when available, to prevent warping. The cases are generally designed to support the rods either flat or on their sides. The rods should not to be leaned against a wall or placed on the damp ground for any extended period, since this can produce a curvature in the rods and result in leveling errors.

i. Rod Levels. When a rod reading is made, it is accurate only if the rod is perfectly plumbed. If the rod is out of plumb, the reading will be greater than the actual vertical distance between the HI and the base of the rod. Therefore, to ensure a truly plumbed leveling rod, a rod level should be used. The two types of rod levels that are generally used with standard leveling rods are shown in Figure 5-20. The one on the left is called the bull's-eye level, and the one on the right is the vial level. Figure 5-21 shows the proper way of attaching the bull's-eye level; the vial level is attached in the same manner.

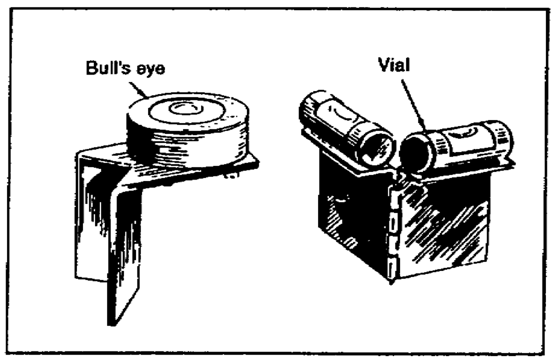


Figure 5-20. Rod levels

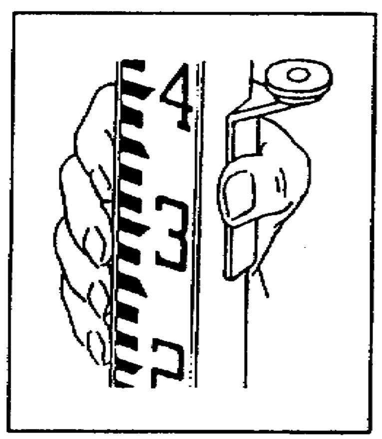


Figure 5-21. Proper attachment of rod level

j. Stadia Boards. In determining linear distance by stadia, you observe a stadia rod or stadia board through a telescope containing stadia hairs and note the size of the interval intercepted by the hairs. A typical stadia board is shown in Figure 5-22. Note that it is graduated in a manner that facilitates counting the number of graduations intercepted between the hairs. Each tenth of a foot is marked by the point of one of the black, saw-toothed graduations. The interval between the point of a black tooth and the next adjacent white gullet between two black teeth represents 0.05 foot.

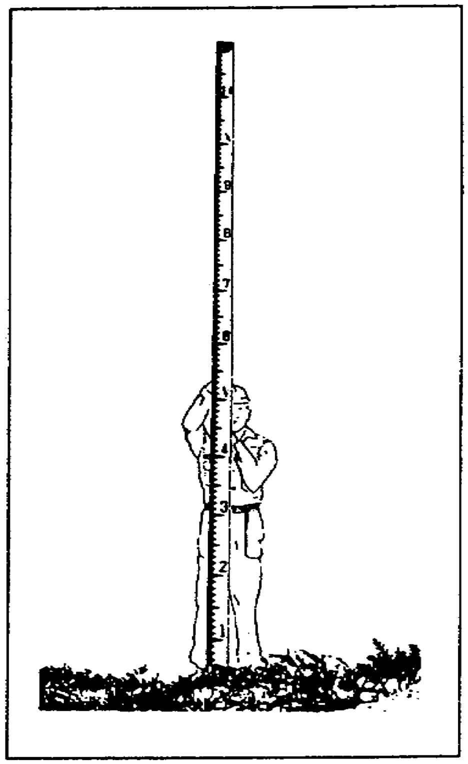


Figure 5-22. Stadia boards

k. Adjusting Pins. Surveying instruments are built in such a way that minor adjustments can be performed in the field without much time loss. The adjustments are made by loosening or tightening the capstan screws with adjusting pins. These pins are included in the instrument box. They come in various sizes, depending on the type of instrument and the hole sizes of its capstan screws. To avoid damage to the head of the capstan screw, use the correct size adjusting pin.

If you lose or break a pin, surveying-equipment dealers will usually replace them free of charge. While conducting a survey, the adjusting pins should be carried in your pocket. This will save valuable time when the pins are needed. Do not use wires, nails, screwdrivers, ink pens, or similar pointed items as substitutes for adjusting pins.

l. Tape Repair Kit. Even though you handle the tape properly and carefully during field measurements, some tapes still break under unforeseen circumstances. During taping operations in the field, the surveyor should always be sure to have an extra tape or a tape repair kit with him so that he can rejoin any broken tape (Figure 5-23). The tape repair kit usually contains a pair of small snips, the tape sections of proper size and graduations, a hand punch or bench punch with block, an assortment of small rivets, a pair of tweezers, a small hammer, and a small file. Before reusing a repaired tape, always check its accuracy by comparing it with another tape that you know is correct.

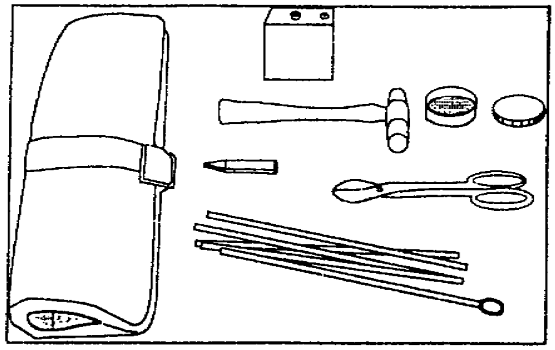


Figure 5-23. Tape repair kit

5-5. Field Supplies. Field supplies consist of a variety of materials used to mark the locations of points in the field. These materials are discussed in the following paragraphs.

a. Surveying Markers. The material used as a survey-point marker depends on where the point is located and whether the marker is to be of a temporary, semipermanent, or permanent character.

(1) Temporary Markers. For purely temporary marking, it is often unnecessary to expend any marking materials. For example, a point in ordinary soil is often temporarily marked by a hole made with the point of a plumb bob, a taping pin, or some other pointed device. In rough taping of distances, even the mere imprint of a heel in the ground may suffice. A point on a concrete surface may be temporarily marked by an X drawn with keel (lumber crayon), a pencil, or some similar marking device. A large nail serves well as a temporary point in relatively stable ground or compacted materials.

(2) Semipermanent Markers. Wooden hubs and stakes are extensively used as semipermanent markers of points in the field. The principal distinction between the two markers is that the top of the hub is usually driven flush, or almost flush, with the ground; whereas with the stake, it is left above the ground several inches.

(a) Wooden hubs are used to mark the station point for an instrument setup. A survey's tack, made of galvanized iron or stainless steel with a depression in the center of the head, is driven into the top of the hub to locate the exact point where the instrument is to be plumbed. Wooden hubs are usually made of a 2- by 2-inch stock and are from 4 to 12 inches long. The average length is about 8 inches; however, shorter lengths can be used in hard ground and longer lengths can be used in soft ground.

(b) Wooden stakes that are improvised in the field may be cylindrical or any other shape that is available. However, manufactured stakes are rectangular in cross section because the faces of the stake are often inscribed with data relevant to the point that the stake is marking. A stake that marks a bench mark, for instance, is inscribed with the symbol that identifies the bench mark and the elevation. A stake that marks a station on a traverse is inscribed with the symbol of the particular station, such as 2 + 45.06. A grade stake is inscribed with the number of vertical feet of cut (material to be excavated) or fill (material to be filled in) required to bring the elevation of the surface to the specified grade elevation. Figure 5-24 shows typical dimensions for an average-sized hub and stake. These dimensions, however, may be modified as situations arise, such as material limitations.

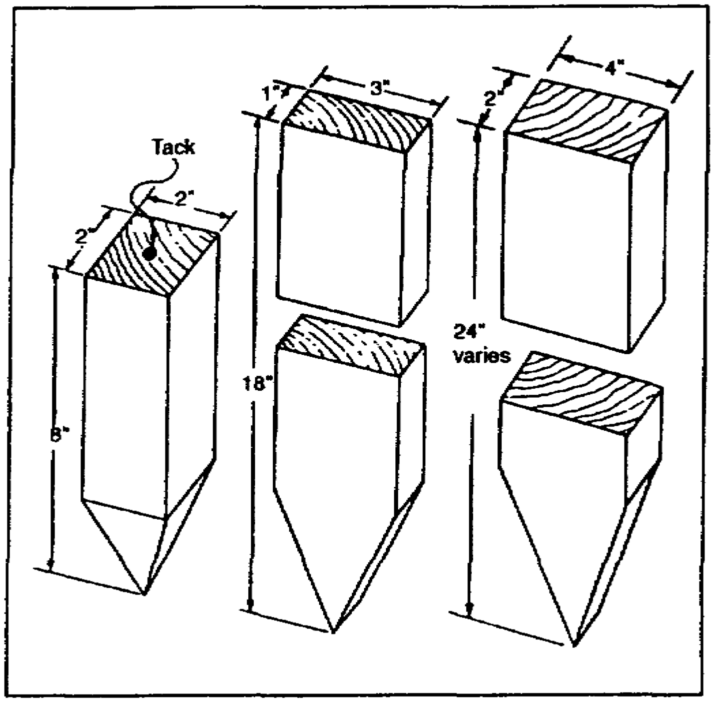


Figure 5-24. Hubs and stakes

(3) Permanent Markers. Permanent markers are used to mark points that are to be used for a long period of time. All permanent markers should be referenced so that they can be replaced if disturbed. Horizontal and vertical control stations are generally marked with permanent markers. These markers could be in the following forms:

(a) Surveyors Tacks, Spikes, and Nails. They are often driven into growing trees, bituminous, or other semisolid surfaces as permanent markers. A nail will be more conspicuous if it is driven through a bottle cap, a washer, a plastic tape, or a "shiner." A shiner is a thin metal disk much like the top or bottom of a frozen fruit-juice can.

(b) Spad. It is a nail equipped with a hook for suspending a plumb bob. It is driven into an overhead surface, such as the top of a tunnel. The suspended plumb bob indicates the point on the floor that is vertically below the spad.

(c) Crosscuts on Existing Concrete Structures or Rock Outcrops. Points on concrete or stone surfaces are often marked with an X by using a hammer and chisel. Another way to do this is to cut holes with a star drill and then plug them with lead.

(d) Metal Pipe. Metal pipe (usually called iron pipe regardless of the actual metal used) runs in lengths of about 18 to 24 inches. Sawed-off lengths of pipe have open ends; pipes cut with a shear have pinched ends and are called pinch pipe. There are also manufactured marker pipes that are T-shaped rather than cylindrical in cross section. A commercial marker may consist of a copper-plated steel rod. All commercial markers have caps or heads that permit center punching for precise point location and stamping of the identifying information.

(e) Concrete Monument. Concrete monuments often have a short length of brass rod set in them to mark the exact location of the point. Federal surveying agencies using concrete monuments as permanent markers set identifying disks in them (see Figure 5-25).

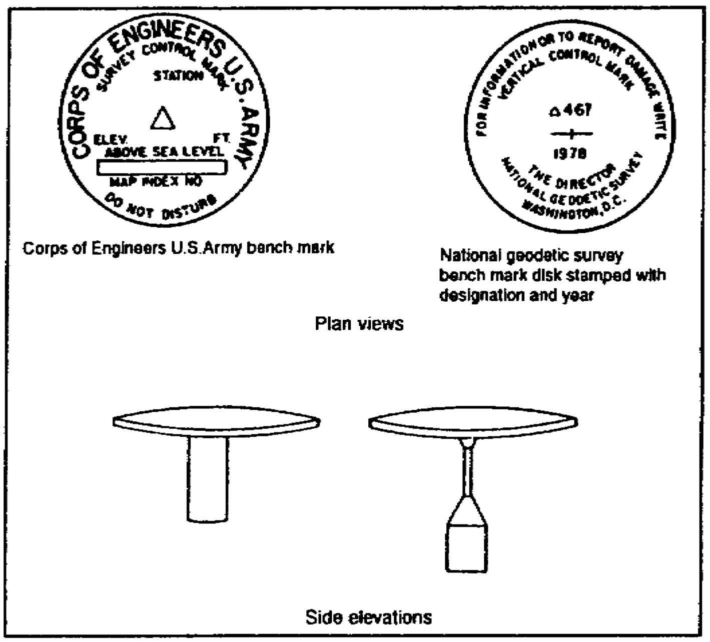


Figure 5-25. Brass Disks

(f) Brass Disk. Manufactured brass disks, similar to the ones shown in Figure 5-25, may be set in grouted holes in street pavements, sidewalks, steps, or the tops of retaining walls.

b. Marking Materials. Keel (lumber crayon) is a thick crayon used for marking stakes or other surfaces. Common marking devices that contain a quick-drying fluid and a felt tip are also popular for marking stakes. All of these types of graphic marking materials come in various colors.

In addition to keel, paint is used to mark pavement surfaces. Paint may be brushed on or sprayed from a spray can. To make the location of a point conspicuous, use a circle, a cross, or a triangle.

Identification symbols, such as station or traverse numbers, may also be painted on. For a neater job, stencils are sometimes used.

c. Flagging. Colored-cloth bunting or plastic tape is often used to make stakes conspicuous so that they will be easier to see. Flagging may also be used for identification purposes. For example, traverse stakes may be marked with one color, grade stakes with another. Red, yellow, orange, and white are the most popular flagging colors.

d. Note-Keeping Materials. Field notes are usually kept in a bound, standard field notebook. Sometimes loose-leaf notebooks are used but are not general recommended because of the chance of losing some pages. In the field notebook, the left-hand side of the page is used for recording measurement data and the right-hand side of the page is used hr remark, sketches, and other supplementary information.

e. Personal Protective and Safety Equipment. In addition to the necessary field supplies and equipment, a field surveying party must carry all the necessary items of personal protective equipment such as containers for drinking water, first-aid kits, gloves, and wet-weather gear, as needed, since they usually work a considerable distance away from the main operational base. For example, if you happen to be taping through a marsh filled with icy water, you would not have a chance to return to the base to get your rubber boots.

In construction areas where the assigned personnel are required to wear hard hats, often, you are also required to wear a hard hat. Be prepared for any situation. Study the situation in advance, considering both the physical and environmental conditions.

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LESSON 5

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. When you have completed the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. What type of viewing magnification does the hand level have?

   A. Two times
   B. Reversed magnification
   C. Short magnification
   D. No magnification

2. How many leveling screws are on the automatic level?

   A. Three
   B. Four
   C. Two
   D. One

3. When should you make use of holes or cracks in the ground for the tripod legs when setting up the tripod?

   A. When the tripod is set up on a hill.
   B. When the tripod is set up on a smooth or slippery surface.
   C. When the tripod is used in conjunction with a range pole.
   D. Every time the tripod is set up.

4. When would it be recommended to use a target on a leveling rod?

   A. When sights are obstructed through leaves and brush.
   B. When a constant grade is needed from one instrument setup.
   C. When there is poor visibility.
   D. All of the above.

5. How is a 2- by 2-inch semipermanent wooden hub placed?

   A. Flush or almost flush with the ground.
   B. As a guard stake.
   C. As the centerline of a road.
   D. As a grade stake.

LESSON 5

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct answer and feedback	
1.	D	No magnification The hand level ... (page 5-6, para 5-1c(3) (b))
2.	A	Three The automatic level has ... (page 5-4, para 5-1c(2))
3.	B	When the tripod is set up on a smooth or slippery surface On smooth or ... (page 5-12, para 5-4a(2)(a))
4.	D	All of the above The leveling rod ... (page 5-16, para 5-4h(1))
5.	A	Flush or almost flush with the ground The principal distinction ... (page 5-20, para 5-5a(2))

APPENDIX A

LIST OF COMMON ACRONYMS

θ	A Greek letter used to identify an angle.
$\sqrt{\ }$	radical sign
"	second
∠	angle
'	minute
+	plus
-	minus
:	ratio
=	equal
___	vinculum
°	degree
∴	therefore
÷	divide
ACCP	Army Correspondence Course Program
AIPD	Army Institute for Professional Development
AISI	automated integrated survey instrument
AMEDD	Army medical department
APO	air post office
app	appendix
AR	Army regulation
ASCE	American Society of Civil Engineers
attn	attention

AV	autovon
AWR	answer weight reference
CDC	Career Development Course
cos	cosine
cot	cotangent
csc	cosecant
DC	District of Columbia
DETC	distance education and training council
DINFOS	Defense Information School
DOD	Department of Defense
EDM	electronic distance measurement
elev	elevation
EN	engineer
EW	east-west
FM	field manual
ft	feet
GD	ground distance
HI	height of instrument
ICE	interservice correspondence exchange
inc	incorporation
IPD	Institute for Professional Development
JFK	John Fitzgerald Kennedy
MD	map distance
mil	A unit of angular measurement equal to 1/6400 of 360˚.

MOS	military occupational specialty
NAVEDTRA	Naval Education Training Aide
no	number
NS	north-south
PX	post exchange
RCOAC	Reserved Component Officer's Advanced Course
RF	representative factor
RS	response sheet
RYE	retirement year ending
see	secant
SGT	sergeant
sin	sine
SS	signal subcourse
SSN	social security number
tan	tangent
TM	technical manual
TRADOC	US Army Training and Doctrine Command
US	United States
VA	Virginia

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APPENDIX B

RECOMMENDED READING LIST

CDC 3E551A. *Engineering Journeyman, Volume 4, Plane Surveying*. Undated.
FM 5-233. *Construction Surveying*. 4 January 1985.
NAVEDTRA 10696. *Engineering Aid 3*. September 1991.
TM 5-232. *Elements of Surveying*. 1 June 1971.

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APPENDIX C

NATURAL TRIGONOMETRIC-FUNCTIONS TABLES

	0 Degree				
Minutes	Sin	Cos	Tan	Cot	
0	0.00000	1.00000	0.00000	#################	60
1	0.00029	1.00000	0.00029	3437.74667	59
2	0.00058	1.00000	0.00058	1718.87319	58
3	0.00087	1.00000	0.00087	1145.91530	57
4	0.00116	1.00000	0.00116	859.43630	56
5	0.00145	1.00000	0.00145	687.54887	55
6	0.00175	1.00000	0.00175	572.95721	54
7	0.00204	1.00000	0.00204	491.10600	53
8	0.00233	1.00000	0.00233	429.71757	52
9	0.00262	1.00000	0.00262	381.97099	51
10	0.00291	1.00000	0.00291	343.77371	50
11	0.00320	0.99999	0.00320	312.52137	49
12	0.00349	0.99999	0.00349	286.47773	48
13	0.00378	0.99999	0.00378	264.44080	47
14	0.00407	0.99999	0.00407	245.55198	46
15	0.00436	0.99999	0.00436	229.18166	45
16	0.00465	0.99999	0.00465	214.85762	44
17	0.00495	0.99999	0.00495	202.21875	43
18	0.00524	0.99999	0.00524	190.98419	42
19	0.00553	0.99998	0.00553	180.93220	41
20	0.00582	0.99998	0.00582	171.88540	40
21	0.00611	0.99998	0.00611	163.70019	39
22	0.00640	0.99998	0.00640	156.25908	38
23	0.00669	0.99998	0.00669	149.46502	37
24	0.00698	0.99998	0.00698	143.23712	36
25	0.00727	0.99997	0.00727	137.50745	35
26	0.00756	0.99997	0.00756	132.21851	34
27	0.00785	0.99997	0.00785	127.32134	33
28	0.00814	0.99997	0.00815	122.77396	32
29	0.00844	0.99996	0.00844	118.54018	31
30	0.00873	0.99996	0.00873	114.58865	30
31	0.00902	0.99996	0.00902	110.89205	29
32	0.00931	0.99996	0.00931	107.42648	28
33	0.00960	0.99995	0.00960	104.17094	27
34	0.00989	0.99995	0.00989	101.10690	26
35	0.01018	0.99995	0.01018	98.21794	25
36	0.01047	0.99995	0.01047	95.48948	24
37	0.01076	0.99994	0.01076	92.90849	23
38	0.01105	0.99994	0.01105	90.46334	22
39	0.01134	0.99994	0.01135	88.14357	21
40	0.01164	0.99993	0.01164	85.93979	20
41	0.01193	0.99993	0.01193	83.84351	19
42	0.01222	0.99993	0.01222	81.84704	18
43	0.01251	0.99992	0.01251	79.94343	17
44	0.01280	0.99992	0.01280	78.12634	16
45	0.01309	0.99991	0.01309	76.39001	15
46	0.01338	0.99991	0.01338	74.72917	14
47	0.01367	0.99991	0.01367	73.13899	13
48	0.01396	0.99990	0.01396	71.61507	12
49	0.01425	0.99990	0.01425	70.15335	11
50	0.01454	0.99989	0.01455	68.75009	10
51	0.01483	0.99989	0.01484	67.40185	9
52	0.01513	0.99989	0.01513	66.10547	8
53	0.01542	0.99988	0.01542	64.85801	7
54	0.01571	0.99988	0.01571	63.65674	6
55	0.01600	0.99987	0.01600	62.49915	5
56	0.01629	0.99987	0.01629	61.38291	4
57	0.01658	0.99986	0.01658	60.30582	3
58	0.01687	0.99986	0.01687	59.26587	2
59	0.01716	0.99985	0.01716	58.26117	1
60	0.01745	0.99985	0.01746	57.28996	0
	Cos	Sin	Cot	Tan	Minutes
			89 Degrees		

1 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.01745	0.99985	0.01746	57.28996	60
1	0.01774	0.99984	0.01775	56.35059	59
2	0.01803	0.99984	0.01804	55.44152	58
3	0.01832	0.99983	0.01833	54.56130	57
4	0.01862	0.99983	0.01862	53.70859	56
5	0.01891	0.99982	0.01891	52.88211	55
6	0.01920	0.99982	0.01920	52.08067	54
7	0.01949	0.99981	0.01949	51.30316	53
8	0.01978	0.99980	0.01978	50.54851	52
9	0.02007	0.99980	0.02007	49.81573	51
10	0.02036	0.99979	0.02036	49.10388	50
11	0.02065	0.99979	0.02066	48.41208	49
12	0.02094	0.99978	0.02095	47.73950	48
13	0.02123	0.99977	0.02124	47.08534	47
14	0.02152	0.99977	0.02153	46.44886	46
15	0.02181	0.99976	0.02182	45.82935	45
16	0.02211	0.99976	0.02211	45.22614	44
17	0.02240	0.99975	0.02240	44.63860	43
18	0.02269	0.99974	0.02269	44.06611	42
19	0.02298	0.99974	0.02298	43.50812	41
20	0.02327	0.99973	0.02328	42.96408	40
21	0.02356	0.99972	0.02357	42.43346	39
22	0.02385	0.99972	0.02386	41.91579	38
23	0.02414	0.99971	0.02415	41.41059	37
24	0.02443	0.99970	0.02444	40.91741	36
25	0.02472	0.99969	0.02473	40.43584	35
26	0.02501	0.99969	0.02502	39.96546	34
27	0.02530	0.99968	0.02531	39.50589	33
28	0.02560	0.99967	0.02560	39.05677	32
29	0.02589	0.99966	0.02589	38.61774	31
30	0.02618	0.99966	0.02619	38.18846	30
31	0.02647	0.99965	0.02648	37.76861	29
32	0.02676	0.99964	0.02677	37.35789	28
33	0.02705	0.99963	0.02706	36.95600	27
34	0.02734	0.99963	0.02735	36.56266	26
35	0.02763	0.99962	0.02764	36.17760	25
36	0.02792	0.99961	0.02793	35.80055	24
37	0.02821	0.99960	0.02822	35.43128	23
38	0.02850	0.99959	0.02851	35.06955	22
39	0.02879	0.99959	0.02881	34.71512	21
40	0.02908	0.99958	0.02910	34.36777	20
41	0.02938	0.99957	0.02939	34.02730	19
42	0.02967	0.99956	0.02968	33.69351	18
43	0.02996	0.99955	0.02997	33.36619	17
44	0.03025	0.99954	0.03026	33.04517	16
45	0.03054	0.99953	0.03055	32.73026	15
46	0.03083	0.99952	0.03084	32.42129	14
47	0.03112	0.99952	0.03114	32.11810	13
48	0.03141	0.99951	0.03143	31.82052	12
49	0.03170	0.99950	0.03172	31.52839	11
50	0.03199	0.99949	0.03201	31.24158	10
51	0.03228	0.99948	0.03230	30.95993	9
52	0.03257	0.99947	0.03259	30.68331	8
53	0.03286	0.99946	0.03288	30.41158	7
54	0.03316	0.99945	0.03317	30.14462	6
55	0.03345	0.99944	0.03346	29.88230	5
56	0.03374	0.99943	0.03376	29.62450	4
57	0.03403	0.99942	0.03405	29.37111	3
58	0.03432	0.99941	0.03434	29.12200	2
59	0.03461	0.99940	0.03463	28.87709	1
60	0.03490	0.99939	0.03492	28.63625	0
	Cos	Sin	Cot	Tan	Minutes

88 Degrees

2 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.03490	0.99939	0.03492	28.63625	60
1	0.03519	0.99938	0.03521	28.39940	59
2	0.03548	0.99937	0.03550	28.16642	58
3	0.03577	0.99936	0.03579	27.93723	57
4	0.03606	0.99935	0.03609	27.71174	56
5	0.03635	0.99934	0.03638	27.48985	55
6	0.03664	0.99933	0.03667	27.27149	54
7	0.03693	0.99932	0.03696	27.05656	53
8	0.03723	0.99931	0.03725	26.84498	52
9	0.03752	0.99930	0.03754	26.63669	51
10	0.03781	0.99929	0.03783	26.43160	50
11	0.03810	0.99927	0.03812	26.22964	49
12	0.03839	0.99926	0.03842	26.03074	48
13	0.03868	0.99925	0.03871	25.83482	47
14	0.03897	0.99924	0.03900	25.64183	46
15	0.03926	0.99923	0.03929	25.45170	45
16	0.03955	0.99922	0.03958	25.26436	44
17	0.03984	0.99921	0.03987	25.07976	43
18	0.04013	0.99919	0.04016	24.89783	42
19	0.04042	0.99918	0.04046	24.71851	41
20	0.04071	0.99917	0.04075	24.54176	40
21	0.04100	0.99916	0.04104	24.36751	39
22	0.04129	0.99915	0.04133	24.19571	38
23	0.04159	0.99913	0.04162	24.02632	37
24	0.04188	0.99912	0.04191	23.85928	36
25	0.04217	0.99911	0.04220	23.69454	35
26	0.04246	0.99910	0.04250	23.53205	34
27	0.04275	0.99909	0.04279	23.37178	33
28	0.04304	0.99907	0.04308	23.21367	32
29	0.04333	0.99906	0.04337	23.05768	31
30	0.04362	0.99905	0.04366	22.90377	30
31	0.04391	0.99904	0.04395	22.75189	29
32	0.04420	0.99902	0.04424	22.60201	28
33	0.04449	0.99901	0.04454	22.45410	27
34	0.04478	0.99900	0.04483	22.30810	26
35	0.04507	0.99898	0.04512	22.16398	25
36	0.04536	0.99897	0.04541	22.02171	24
37	0.04565	0.99896	0.04570	21.88125	23
38	0.04594	0.99894	0.04599	21.74257	22
39	0.04623	0.99893	0.04628	21.60563	21
40	0.04653	0.99892	0.04658	21.47040	20
41	0.04682	0.99890	0.04687	21.33685	19
42	0.04711	0.99889	0.04716	21.20495	18
43	0.04740	0.99888	0.04745	21.07466	17
44	0.04769	0.99886	0.04774	20.94597	16
45	0.04798	0.99885	0.04803	20.81883	15
46	0.04827	0.99883	0.04833	20.69322	14
47	0.04856	0.99882	0.04862	20.56911	13
48	0.04885	0.99881	0.04891	20.44649	12
49	0.04914	0.99879	0.04920	20.32531	11
50	0.04943	0.99878	0.04949	20.20555	10
51	0.04972	0.99876	0.04978	20.08720	9
52	0.05001	0.99875	0.05007	19.97022	8
53	0.05030	0.99873	0.05037	19.85459	7
54	0.05059	0.99872	0.05066	19.74029	6
55	0.05088	0.99870	0.05095	19.62730	5
56	0.05117	0.99869	0.05124	19.51558	4
57	0.05146	0.99867	0.05153	19.40513	3
58	0.05175	0.99866	0.05182	19.29592	2
59	0.05205	0.99864	0.05212	19.18793	1
60	0.05234	0.99863	0.05241	19.08114	0
	Cos	Sin	Cot	Tan	Minutes

87 Degrees

3 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.05234	0.99863	0.05241	19.08114	60
1	0.05263	0.99861	0.05270	18.97552	59
2	0.05292	0.99860	0.05299	18.87107	58
3	0.05321	0.99858	0.05328	18.76775	57
4	0.05350	0.99857	0.05357	18.66556	56
5	0.05379	0.99855	0.05387	18.56447	55
6	0.05408	0.99854	0.05416	18.46447	54
7	0.05437	0.99852	0.05445	18.36554	53
8	0.05466	0.99851	0.05474	18.26765	52
9	0.05495	0.99849	0.05503	18.17081	51
10	0.05524	0.99847	0.05533	18.07498	50
11	0.05553	0.99846	0.05562	17.98015	49
12	0.05582	0.99844	0.05591	17.88631	48
13	0.05611	0.99842	0.05620	17.79344	47
14	0.05640	0.99841	0.05649	17.70153	46
15	0.05669	0.99839	0.05678	17.61056	45
16	0.05698	0.99838	0.05708	17.52052	44
17	0.05727	0.99836	0.05737	17.43139	43
18	0.05756	0.99834	0.05766	17.34315	42
19	0.05785	0.99833	0.05795	17.25581	41
20	0.05814	0.99831	0.05824	17.16934	40
21	0.05844	0.99829	0.05854	17.08372	39
22	0.05873	0.99827	0.05883	16.99896	38
23	0.05902	0.99826	0.05912	16.91503	37
24	0.05931	0.99824	0.05941	16.83191	36
25	0.05960	0.99822	0.05970	16.74961	35
26	0.05989	0.99821	0.05999	16.66811	34
27	0.06018	0.99819	0.06029	16.58740	33
28	0.06047	0.99817	0.06058	16.50746	32
29	0.06076	0.99815	0.06087	16.42828	31
30	0.06105	0.99813	0.06116	16.34986	30
31	0.06134	0.99812	0.06145	16.27217	29
32	0.06163	0.99810	0.06175	16.19523	28
33	0.06192	0.99808	0.06204	16.11900	27
34	0.06221	0.99806	0.06233	16.04348	26
35	0.06250	0.99804	0.06262	15.96867	25
36	0.06279	0.99803	0.06291	15.89454	24
37	0.06308	0.99801	0.06321	15.82110	23
38	0.06337	0.99799	0.06350	15.74834	22
39	0.06366	0.99797	0.06379	15.67623	21
40	0.06395	0.99795	0.06408	15.60478	20
41	0.06424	0.99793	0.06438	15.53398	19
42	0.06453	0.99792	0.06467	15.46381	18
43	0.06482	0.99790	0.06496	15.39428	17
44	0.06511	0.99788	0.06525	15.32536	16
45	0.06540	0.99786	0.06554	15.25705	15
46	0.06569	0.99784	0.06584	15.18935	14
47	0.06598	0.99782	0.06613	15.12224	13
48	0.06627	0.99780	0.06642	15.05572	12
49	0.06656	0.99778	0.06671	14.98978	11
50	0.06685	0.99776	0.06700	14.92442	10
51	0.06714	0.99774	0.06730	14.85962	9
52	0.06743	0.99772	0.06759	14.79537	8
53	0.06773	0.99770	0.06788	14.73168	7
54	0.06802	0.99768	0.06817	14.66853	6
55	0.06831	0.99766	0.06847	14.60592	5
56	0.06860	0.99764	0.06876	14.54383	4
57	0.06889	0.99762	0.06905	14.48227	3
58	0.06918	0.99760	0.06934	14.42123	2
59	0.06947	0.99758	0.06963	14.36070	1
60	0.06976	0.99756	0.06993	14.30067	0
	Cos	Sin	Cot	Tan	Minutes

86 Degrees

4 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.06976	0.99756	0.06993	14.30067	60
1	0.07005	0.99754	0.07022	14.24113	59
2	0.07034	0.99752	0.07051	14.18209	58
3	0.07063	0.99750	0.07080	14.12354	57
4	0.07092	0.99748	0.07110	14.06546	56
5	0.07121	0.99746	0.07139	14.00786	55
6	0.07150	0.99744	0.07168	13.95072	54
7	0.07179	0.99742	0.07197	13.89405	53
8	0.07208	0.99740	0.07227	13.83783	52
9	0.07237	0.99738	0.07256	13.78206	51
10	0.07266	0.99736	0.07285	13.72674	50
11	0.07295	0.99734	0.07314	13.67186	49
12	0.07324	0.99731	0.07344	13.61741	48
13	0.07353	0.99729	0.07373	13.56339	47
14	0.07382	0.99727	0.07402	13.50980	46
15	0.07411	0.99725	0.07431	13.45663	45
16	0.07440	0.99723	0.07461	13.40387	44
17	0.07469	0.99721	0.07490	13.35152	43
18	0.07498	0.99719	0.07519	13.29957	42
19	0.07527	0.99716	0.07548	13.24803	41
20	0.07556	0.99714	0.07578	13.19688	40
21	0.07585	0.99712	0.07607	13.14613	39
22	0.07614	0.99710	0.07636	13.09576	38
23	0.07643	0.99708	0.07665	13.04577	37
24	0.07672	0.99705	0.07695	12.99616	36
25	0.07701	0.99703	0.07724	12.94692	35
26	0.07730	0.99701	0.07753	12.89806	34
27	0.07759	0.99699	0.07782	12.84956	33
28	0.07788	0.99696	0.07812	12.80142	32
29	0.07817	0.99694	0.07841	12.75363	31
30	0.07846	0.99692	0.07870	12.70620	30
31	0.07875	0.99689	0.07899	12.65912	29
32	0.07904	0.99687	0.07929	12.61239	28
33	0.07933	0.99685	0.07958	12.56600	27
34	0.07962	0.99683	0.07987	12.51994	26
35	0.07991	0.99680	0.08017	12.47422	25
36	0.08020	0.99678	0.08046	12.42883	24
37	0.08049	0.99676	0.08075	12.38377	23
38	0.08078	0.99673	0.08104	12.33903	22
39	0.08107	0.99671	0.08134	12.29461	21
40	0.08136	0.99668	0.08163	12.25051	20
41	0.08165	0.99666	0.08192	12.20672	19
42	0.08194	0.99664	0.08221	12.16324	18
43	0.08223	0.99661	0.08251	12.12006	17
44	0.08252	0.99659	0.08280	12.07719	16
45	0.08281	0.99657	0.08309	12.03462	15
46	0.08310	0.99654	0.08339	11.99235	14
47	0.08339	0.99652	0.08368	11.95037	13
48	0.08368	0.99649	0.08397	11.90868	12
49	0.08397	0.99647	0.08427	11.86728	11
50	0.08426	0.99644	0.08456	11.82617	10
51	0.08455	0.99642	0.08485	11.78533	9
52	0.08484	0.99639	0.08514	11.74478	8
53	0.08513	0.99637	0.08544	11.70450	7
54	0.08542	0.99635	0.08573	11.66450	6
55	0.08571	0.99632	0.08602	11.62476	5
56	0.08600	0.99630	0.08632	11.58529	4
57	0.08629	0.99627	0.08661	11.54609	3
58	0.08658	0.99625	0.08690	11.50715	2
59	0.08687	0.99622	0.08720	11.46847	1
60	0.08716	0.99619	0.08749	11.43005	0
	Cos	Sin	Cot	Tan	Minutes

85 Degrees

5 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.08716	0.99619	0.08749	11.43005	60
1	0.08745	0.99617	0.08778	11.39188	59
2	0.08774	0.99614	0.08807	11.35397	58
3	0.08803	0.99612	0.08837	11.31630	57
4	0.08831	0.99609	0.08866	11.27889	56
5	0.08860	0.99607	0.08895	11.24171	55
6	0.08889	0.99604	0.08925	11.20478	54
7	0.08918	0.99602	0.08954	11.16809	53
8	0.08947	0.99599	0.08983	11.13163	52
9	0.08976	0.99596	0.09013	11.09542	51
10	0.09005	0.99594	0.09042	11.05943	50
11	0.09034	0.99591	0.09071	11.02368	49
12	0.09063	0.99588	0.09101	10.98815	48
13	0.09092	0.99586	0.09130	10.95285	47
14	0.09121	0.99583	0.09159	10.91777	46
15	0.09150	0.99580	0.09189	10.88292	45
16	0.09179	0.99578	0.09218	10.84829	44
17	0.09208	0.99575	0.09247	10.81387	43
18	0.09237	0.99572	0.09277	10.77967	42
19	0.09266	0.99570	0.09306	10.74569	41
20	0.09295	0.99567	0.09335	10.71191	40
21	0.09324	0.99564	0.09365	10.67835	39
22	0.09353	0.99562	0.09394	10.64499	38
23	0.09382	0.99559	0.09423	10.61184	37
24	0.09411	0.99556	0.09453	10.57889	36
25	0.09440	0.99553	0.09482	10.54615	35
26	0.09469	0.99551	0.09511	10.51361	34
27	0.09498	0.99548	0.09541	10.48126	33
28	0.09527	0.99545	0.09570	10.44911	32
29	0.09556	0.99542	0.09600	10.41716	31
30	0.09585	0.99540	0.09629	10.38540	30
31	0.09614	0.99537	0.09658	10.35383	29
32	0.09642	0.99534	0.09688	10.32245	28
33	0.09671	0.99531	0.09717	10.29126	27
34	0.09700	0.99528	0.09746	10.26025	26
35	0.09729	0.99526	0.09776	10.22943	25
36	0.09758	0.99523	0.09805	10.19879	24
37	0.09787	0.99520	0.09834	10.16833	23
38	0.09816	0.99517	0.09864	10.13805	22
39	0.09845	0.99514	0.09893	10.10795	21
40	0.09874	0.99511	0.09923	10.07803	20
41	0.09903	0.99508	0.09952	10.04828	19
42	0.09932	0.99506	0.09981	10.01871	18
43	0.09961	0.99503	0.10011	9.98931	17
44	0.09990	0.99500	0.10040	9.96007	16
45	0.10019	0.99497	0.10069	9.93101	15
46	0.10048	0.99494	0.10099	9.90211	14
47	0.10077	0.99491	0.10128	9.87338	13
48	0.10106	0.99488	0.10158	9.84482	12
49	0.10135	0.99485	0.10187	9.81641	11
50	0.10164	0.99482	0.10216	9.78817	10
51	0.10192	0.99479	0.10246	9.76009	9
52	0.10221	0.99476	0.10275	9.73217	8
53	0.10250	0.99473	0.10305	9.70441	7
54	0.10279	0.99470	0.10334	9.67680	6
55	0.10308	0.99467	0.10363	9.64935	5
56	0.10337	0.99464	0.10393	9.62205	4
57	0.10366	0.99461	0.10422	9.59490	3
58	0.10395	0.99458	0.10452	9.56791	2
59	0.10424	0.99455	0.10481	9.54106	1
60	0.10453	0.99452	0.10510	9.51436	0
	Cos	Sin	Cot	Tan	Minutes

84 Degrees

6 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.10453	0.99452	0.10510	9.51436	60
1	0.10482	0.99449	0.10540	9.48781	59
2	0.10511	0.99446	0.10569	9.46141	58
3	0.10540	0.99443	0.10599	9.43515	57
4	0.10569	0.99440	0.10628	9.40904	56
5	0.10597	0.99437	0.10657	9.38307	55
6	0.10626	0.99434	0.10687	9.35724	54
7	0.10655	0.99431	0.10716	9.33155	53
8	0.10684	0.99428	0.10746	9.30599	52
9	0.10713	0.99424	0.10775	9.28058	51
10	0.10742	0.99421	0.10805	9.25530	50
11	0.10771	0.99418	0.10834	9.23016	49
12	0.10800	0.99415	0.10863	9.20516	48
13	0.10829	0.99412	0.10893	9.18028	47
14	0.10858	0.99409	0.10922	9.15554	46
15	0.10887	0.99406	0.10952	9.13093	45
16	0.10916	0.99402	0.10981	9.10646	44
17	0.10945	0.99399	0.11011	9.08211	43
18	0.10973	0.99396	0.11040	9.05789	42
19	0.11002	0.99393	0.11070	9.03379	41
20	0.11031	0.99390	0.11099	9.00983	40
21	0.11060	0.99386	0.11128	8.98598	39
22	0.11089	0.99383	0.11158	8.96227	38
23	0.11118	0.99380	0.11187	8.93867	37
24	0.11147	0.99377	0.11217	8.91520	36
25	0.11176	0.99374	0.11246	8.89185	35
26	0.11205	0.99370	0.11276	8.86862	34
27	0.11234	0.99367	0.11305	8.84551	33
28	0.11263	0.99364	0.11335	8.82252	32
29	0.11291	0.99360	0.11364	8.79964	31
30	0.11320	0.99357	0.11394	8.77689	30
31	0.11349	0.99354	0.11423	8.75425	29
32	0.11378	0.99351	0.11452	8.73172	28
33	0.11407	0.99347	0.11482	8.70931	27
34	0.11436	0.99344	0.11511	8.68701	26
35	0.11465	0.99341	0.11541	8.66482	25
36	0.11494	0.99337	0.11570	8.64275	24
37	0.11523	0.99334	0.11600	8.62078	23
38	0.11552	0.99331	0.11629	8.59893	22
39	0.11580	0.99327	0.11659	8.57718	21
40	0.11609	0.99324	0.11688	8.55555	20
41	0.11638	0.99320	0.11718	8.53402	19
42	0.11667	0.99317	0.11747	8.51259	18
43	0.11696	0.99314	0.11777	8.49128	17
44	0.11725	0.99310	0.11806	8.47007	16
45	0.11754	0.99307	0.11836	8.44896	15
46	0.11783	0.99303	0.11865	8.42795	14
47	0.11812	0.99300	0.11895	8.40705	13
48	0.11840	0.99297	0.11924	8.38625	12
49	0.11869	0.99293	0.11954	8.36555	11
50	0.11898	0.99290	0.11983	8.34496	10
51	0.11927	0.99286	0.12013	8.32446	9
52	0.11956	0.99283	0.12042	8.30406	8
53	0.11985	0.99279	0.12072	8.28376	7
54	0.12014	0.99276	0.12101	8.26355	6
55	0.12043	0.99272	0.12131	8.24345	5
56	0.12071	0.99269	0.12160	8.22344	4
57	0.12100	0.99265	0.12190	8.20352	3
58	0.12129	0.99262	0.12219	8.18370	2
59	0.12158	0.99258	0.12249	8.16398	1
60	0.12187	0.99255	0.12278	8.14435	0
	Cos	Sin	Cot	Tan	Minutes

83 Degrees

7 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.12187	0.99255	0.12278	8.14435	60
1	0.12216	0.99251	0.12308	8.12481	59
2	0.12245	0.99248	0.12338	8.10536	58
3	0.12274	0.99244	0.12367	8.08600	57
4	0.12302	0.99240	0.12397	8.06674	56
5	0.12331	0.99237	0.12426	8.04756	55
6	0.12360	0.99233	0.12456	8.02848	54
7	0.12389	0.99230	0.12485	8.00948	53
8	0.12418	0.99226	0.12515	7.99058	52
9	0.12447	0.99222	0.12544	7.97176	51
10	0.12476	0.99219	0.12574	7.95302	50
11	0.12504	0.99215	0.12603	7.93438	49
12	0.12533	0.99211	0.12633	7.91582	48
13	0.12562	0.99208	0.12662	7.89734	47
14	0.12591	0.99204	0.12692	7.87895	46
15	0.12620	0.99200	0.12722	7.86064	45
16	0.12649	0.99197	0.12751	7.84242	44
17	0.12678	0.99193	0.12781	7.82428	43
18	0.12706	0.99189	0.12810	7.80622	42
19	0.12735	0.99186	0.12840	7.78825	41
20	0.12764	0.99182	0.12869	7.77035	40
21	0.12793	0.99178	0.12899	7.75254	39
22	0.12822	0.99175	0.12929	7.73480	38
23	0.12851	0.99171	0.12958	7.71715	37
24	0.12880	0.99167	0.12988	7.69957	36
25	0.12908	0.99163	0.13017	7.68208	35
26	0.12937	0.99160	0.13047	7.66466	34
27	0.12966	0.99156	0.13076	7.64732	33
28	0.12995	0.99152	0.13106	7.63005	32
29	0.13024	0.99148	0.13136	7.61287	31
30	0.13053	0.99144	0.13165	7.59575	30
31	0.13081	0.99141	0.13195	7.57872	29
32	0.13110	0.99137	0.13224	7.56176	28
33	0.13139	0.99133	0.13254	7.54487	27
34	0.13168	0.99129	0.13284	7.52806	26
35	0.13197	0.99125	0.13313	7.51132	25
36	0.13226	0.99122	0.13343	7.49465	24
37	0.13254	0.99118	0.13372	7.47806	23
38	0.13283	0.99114	0.13402	7.46154	22
39	0.13312	0.99110	0.13432	7.44509	21
40	0.13341	0.99106	0.13461	7.42871	20
41	0.13370	0.99102	0.13491	7.41240	19
42	0.13399	0.99098	0.13521	7.39616	18
43	0.13427	0.99094	0.13550	7.37999	17
44	0.13456	0.99091	0.13580	7.36389	16
45	0.13485	0.99087	0.13609	7.34786	15
46	0.13514	0.99083	0.13639	7.33190	14
47	0.13543	0.99079	0.13669	7.31600	13
48	0.13572	0.99075	0.13698	7.30018	12
49	0.13600	0.99071	0.13728	7.28442	11
50	0.13629	0.99067	0.13758	7.26873	10
51	0.13658	0.99063	0.13787	7.25310	9
52	0.13687	0.99059	0.13817	7.23754	8
53	0.13716	0.99055	0.13846	7.22204	7
54	0.13744	0.99051	0.13876	7.20661	6
55	0.13773	0.99047	0.13906	7.19125	5
56	0.13802	0.99043	0.13935	7.17594	4
57	0.13831	0.99039	0.13965	7.16071	3
58	0.13860	0.99035	0.13995	7.14553	2
59	0.13889	0.99031	0.14024	7.13042	1
60	0.13917	0.99027	0.14054	7.11537	0
	Cos	Sin	Cot	Tan	Minutes

82 Degrees

8 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.13917	0.99027	0.14054	7.11537	60
1	0.13946	0.99023	0.14084	7.10038	59
2	0.13975	0.99019	0.14113	7.08546	58
3	0.14004	0.99015	0.14143	7.07059	57
4	0.14033	0.99011	0.14173	7.05579	56
5	0.14061	0.99006	0.14202	7.04105	55
6	0.14090	0.99002	0.14232	7.02637	54
7	0.14119	0.98998	0.14262	7.01174	53
8	0.14148	0.98994	0.14291	6.99718	52
9	0.14177	0.98990	0.14321	6.98268	51
10	0.14205	0.98986	0.14351	6.96823	50
11	0.14234	0.98982	0.14381	6.95385	49
12	0.14263	0.98978	0.14410	6.93952	48
13	0.14292	0.98973	0.14440	6.92525	47
14	0.14320	0.98969	0.14470	6.91104	46
15	0.14349	0.98965	0.14499	6.89688	45
16	0.14378	0.98961	0.14529	6.88278	44
17	0.14407	0.98957	0.14559	6.86874	43
18	0.14436	0.98953	0.14588	6.85475	42
19	0.14464	0.98948	0.14618	6.84082	41
20	0.14493	0.98944	0.14648	6.82694	40
21	0.14522	0.98940	0.14678	6.81312	39
22	0.14551	0.98936	0.14707	6.79936	38
23	0.14580	0.98931	0.14737	6.78564	37
24	0.14608	0.98927	0.14767	6.77199	36
25	0.14637	0.98923	0.14796	6.75838	35
26	0.14666	0.98919	0.14826	6.74483	34
27	0.14695	0.98914	0.14856	6.73133	33
28	0.14723	0.98910	0.14886	6.71789	32
29	0.14752	0.98906	0.14915	6.70450	31
30	0.14781	0.98902	0.14945	6.69116	30
31	0.14810	0.98897	0.14975	6.67787	29
32	0.14838	0.98893	0.15005	6.66463	28
33	0.14867	0.98889	0.15034	6.65144	27
34	0.14896	0.98884	0.15064	6.63831	26
35	0.14925	0.98880	0.15094	6.62523	25
36	0.14954	0.98876	0.15124	6.61219	24
37	0.14982	0.98871	0.15153	6.59921	23
38	0.15011	0.98867	0.15183	6.58627	22
39	0.15040	0.98863	0.15213	6.57339	21
40	0.15069	0.98858	0.15243	6.56055	20
41	0.15097	0.98854	0.15272	6.54777	19
42	0.15126	0.98849	0.15302	6.53503	18
43	0.15155	0.98845	0.15332	6.52234	17
44	0.15184	0.98841	0.15362	6.50970	16
45	0.15212	0.98836	0.15391	6.49710	15
46	0.15241	0.98832	0.15421	6.48456	14
47	0.15270	0.98827	0.15451	6.47206	13
48	0.15299	0.98823	0.15481	6.45961	12
49	0.15327	0.98818	0.15511	6.44720	11
50	0.15356	0.98814	0.15540	6.43484	10
51	0.15385	0.98809	0.15570	6.42253	9
52	0.15414	0.98805	0.15600	6.41026	8
53	0.15442	0.98800	0.15630	6.39804	7
54	0.15471	0.98796	0.15660	6.38587	6
55	0.15500	0.98791	0.15689	6.37374	5
56	0.15529	0.98787	0.15719	6.36165	4
57	0.15557	0.98782	0.15749	6.34961	3
58	0.15586	0.98778	0.15779	6.33761	2
59	0.15615	0.98773	0.15809	6.32566	1
60	0.15643	0.98769	0.15838	6.31375	0
	Cos	Sin	Cot	Tan	Minutes

81 Degrees

9 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.15643	0.98769	0.15838	6.31375	60
1	0.15672	0.98764	0.15868	6.30189	59
2	0.15701	0.98760	0.15898	6.29007	58
3	0.15730	0.98755	0.15928	6.27829	57
4	0.15758	0.98751	0.15958	6.26655	56
5	0.15787	0.98746	0.15988	6.25486	55
6	0.15816	0.98741	0.16017	6.24321	54
7	0.15845	0.98737	0.16047	6.23160	53
8	0.15873	0.98732	0.16077	6.22003	52
9	0.15902	0.98728	0.16107	6.20851	51
10	0.15931	0.98723	0.16137	6.19703	50
11	0.15959	0.98718	0.16167	6.18559	49
12	0.15988	0.98714	0.16196	6.17419	48
13	0.16017	0.98709	0.16226	6.16283	47
14	0.16046	0.98704	0.16256	6.15151	46
15	0.16074	0.98700	0.16286	6.14023	45
16	0.16103	0.98695	0.16316	6.12899	44
17	0.16132	0.98690	0.16346	6.11779	43
18	0.16160	0.98686	0.16376	6.10664	42
19	0.16189	0.98681	0.16405	6.09552	41
20	0.16218	0.98676	0.16435	6.08444	40
21	0.16246	0.98671	0.16465	6.07340	39
22	0.16275	0.98667	0.16495	6.06240	38
23	0.16304	0.98662	0.16525	6.05143	37
24	0.16333	0.98657	0.16555	6.04051	36
25	0.16361	0.98652	0.16585	6.02962	35
26	0.16390	0.98648	0.16615	6.01878	34
27	0.16419	0.98643	0.16645	6.00797	33
28	0.16447	0.98638	0.16674	5.99720	32
29	0.16476	0.98633	0.16704	5.98646	31
30	0.16505	0.98629	0.16734	5.97576	30
31	0.16533	0.98624	0.16764	5.96510	29
32	0.16562	0.98619	0.16794	5.95448	28
33	0.16591	0.98614	0.16824	5.94390	27
34	0.16620	0.98609	0.16854	5.93335	26
35	0.16648	0.98604	0.16884	5.92283	25
36	0.16677	0.98600	0.16914	5.91236	24
37	0.16706	0.98595	0.16944	5.90191	23
38	0.16734	0.98590	0.16974	5.89151	22
39	0.16763	0.98585	0.17004	5.88114	21
40	0.16792	0.98580	0.17033	5.87080	20
41	0.16820	0.98575	0.17063	5.86051	19
42	0.16849	0.98570	0.17093	5.85024	18
43	0.16878	0.98565	0.17123	5.84001	17
44	0.16906	0.98561	0.17153	5.82982	16
45	0.16935	0.98556	0.17183	5.81966	15
46	0.16964	0.98551	0.17213	5.80953	14
47	0.16992	0.98546	0.17243	5.79944	13
48	0.17021	0.98541	0.17273	5.78938	12
49	0.17050	0.98536	0.17303	5.77936	11
50	0.17078	0.98531	0.17333	5.76937	10
51	0.17107	0.98526	0.17363	5.75941	9
52	0.17136	0.98521	0.17393	5.74949	8
53	0.17164	0.98516	0.17423	5.73960	7
54	0.17193	0.98511	0.17453	5.72974	6
55	0.17222	0.98506	0.17483	5.71992	5
56	0.17250	0.98501	0.17513	5.71013	4
57	0.17279	0.98496	0.17543	5.70037	3
58	0.17308	0.98491	0.17573	5.69064	2
59	0.17336	0.98486	0.17603	5.68094	1
60	0.17365	0.98481	0.17633	5.67128	0
	Cos	Sin	Cot	Tan	Minutes

80 Degrees

10 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.17365	0.98481	0.17633	5.67128	60
1	0.17393	0.98476	0.17663	5.66165	59
2	0.17422	0.98471	0.17693	5.65205	58
3	0.17451	0.98466	0.17723	5.64248	57
4	0.17479	0.98461	0.17753	5.63295	56
5	0.17508	0.98455	0.17783	5.62344	55
6	0.17537	0.98450	0.17813	5.61397	54
7	0.17565	0.98445	0.17843	5.60452	53
8	0.17594	0.98440	0.17873	5.59511	52
9	0.17623	0.98435	0.17903	5.58573	51
10	0.17651	0.98430	0.17933	5.57638	50
11	0.17680	0.98425	0.17963	5.56706	49
12	0.17708	0.98420	0.17993	5.55777	48
13	0.17737	0.98414	0.18023	5.54851	47
14	0.17766	0.98409	0.18053	5.53927	46
15	0.17794	0.98404	0.18083	5.53007	45
16	0.17823	0.98399	0.18113	5.52090	44
17	0.17852	0.98394	0.18143	5.51176	43
18	0.17880	0.98389	0.18173	5.50264	42
19	0.17909	0.98383	0.18203	5.49356	41
20	0.17937	0.98378	0.18233	5.48451	40
21	0.17966	0.98373	0.18263	5.47548	39
22	0.17995	0.98368	0.18293	5.46648	38
23	0.18023	0.98362	0.18323	5.45751	37
24	0.18052	0.98357	0.18353	5.44857	36
25	0.18081	0.98352	0.18384	5.43966	35
26	0.18109	0.98347	0.18414	5.43077	34
27	0.18138	0.98341	0.18444	5.42192	33
28	0.18166	0.98336	0.18474	5.41309	32
29	0.18195	0.98331	0.18504	5.40429	31
30	0.18224	0.98325	0.18534	5.39552	30
31	0.18252	0.98320	0.18564	5.38677	29
32	0.18281	0.98315	0.18594	5.37805	28
33	0.18309	0.98310	0.18624	5.36936	27
34	0.18338	0.98304	0.18654	5.36070	26
35	0.18367	0.98299	0.18684	5.35206	25
36	0.18395	0.98294	0.18714	5.34345	24
37	0.18424	0.98288	0.18745	5.33487	23
38	0.18452	0.98283	0.18775	5.32631	22
39	0.18481	0.98277	0.18805	5.31778	21
40	0.18509	0.98272	0.18835	5.30928	20
41	0.18538	0.98267	0.18865	5.30080	19
42	0.18567	0.98261	0.18895	5.29235	18
43	0.18595	0.98256	0.18925	5.28393	17
44	0.18624	0.98250	0.18955	5.27553	16
45	0.18652	0.98245	0.18986	5.26715	15
46	0.18681	0.98240	0.19016	5.25880	14
47	0.18710	0.98234	0.19046	5.25048	13
48	0.18738	0.98229	0.19076	5.24218	12
49	0.18767	0.98223	0.19106	5.23391	11
50	0.18795	0.98218	0.19136	5.22566	10
51	0.18824	0.98212	0.19166	5.21744	9
52	0.18852	0.98207	0.19197	5.20925	8
53	0.18881	0.98201	0.19227	5.20107	7
54	0.18910	0.98196	0.19257	5.19293	6
55	0.18938	0.98190	0.19287	5.18480	5
56	0.18967	0.98185	0.19317	5.17671	4
57	0.18995	0.98179	0.19347	5.16863	3
58	0.19024	0.98174	0.19378	5.16058	2
59	0.19052	0.98168	0.19408	5.15256	1
60	0.19081	0.98163	0.19438	5.14455	0
	Cos	Sin	Cot	Tan	Minutes

79 Degrees

11 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.19081	0.98163	0.19438	5.14455	60
1	0.19109	0.98157	0.19468	5.13658	59
2	0.19138	0.98152	0.19498	5.12862	58
3	0.19167	0.98146	0.19529	5.12069	57
4	0.19195	0.98140	0.19559	5.11279	56
5	0.19224	0.98135	0.19589	5.10490	55
6	0.19252	0.98129	0.19619	5.09704	54
7	0.19281	0.98124	0.19649	5.08921	53
8	0.19309	0.98118	0.19680	5.08139	52
9	0.19338	0.98112	0.19710	5.07360	51
10	0.19366	0.98107	0.19740	5.06584	50
11	0.19395	0.98101	0.19770	5.05809	49
12	0.19423	0.98096	0.19801	5.05037	48
13	0.19452	0.98090	0.19831	5.04267	47
14	0.19481	0.98084	0.19861	5.03499	46
15	0.19509	0.98079	0.19891	5.02734	45
16	0.19538	0.98073	0.19921	5.01971	44
17	0.19566	0.98067	0.19952	5.01210	43
18	0.19595	0.98061	0.19982	5.00451	42
19	0.19623	0.98056	0.20012	4.99695	41
20	0.19652	0.98050	0.20042	4.98940	40
21	0.19680	0.98044	0.20073	4.98188	39
22	0.19709	0.98039	0.20103	4.97438	38
23	0.19737	0.98033	0.20133	4.96690	37
24	0.19766	0.98027	0.20164	4.95945	36
25	0.19794	0.98021	0.20194	4.95201	35
26	0.19823	0.98016	0.20224	4.94460	34
27	0.19851	0.98010	0.20254	4.93721	33
28	0.19880	0.98004	0.20285	4.92984	32
29	0.19908	0.97998	0.20315	4.92249	31
30	0.19937	0.97992	0.20345	4.91516	30
31	0.19965	0.97987	0.20376	4.90785	29
32	0.19994	0.97981	0.20406	4.90056	28
33	0.20022	0.97975	0.20436	4.89330	27
34	0.20051	0.97969	0.20466	4.88605	26
35	0.20079	0.97963	0.20497	4.87882	25
36	0.20108	0.97958	0.20527	4.87162	24
37	0.20136	0.97952	0.20557	4.86444	23
38	0.20165	0.97946	0.20588	4.85727	22
39	0.20193	0.97940	0.20618	4.85013	21
40	0.20222	0.97934	0.20648	4.84300	20
41	0.20250	0.97928	0.20679	4.83590	19
42	0.20279	0.97922	0.20709	4.82882	18
43	0.20307	0.97916	0.20739	4.82175	17
44	0.20336	0.97910	0.20770	4.81471	16
45	0.20364	0.97905	0.20800	4.80769	15
46	0.20393	0.97899	0.20830	4.80068	14
47	0.20421	0.97893	0.20861	4.79370	13
48	0.20450	0.97887	0.20891	4.78673	12
49	0.20478	0.97881	0.20921	4.77978	11
50	0.20507	0.97875	0.20952	4.77286	10
51	0.20535	0.97869	0.20982	4.76595	9
52	0.20563	0.97863	0.21013	4.75906	8
53	0.20592	0.97857	0.21043	4.75219	7
54	0.20620	0.97851	0.21073	4.74534	6
55	0.20649	0.97845	0.21104	4.73851	5
56	0.20677	0.97839	0.21134	4.73170	4
57	0.20706	0.97833	0.21164	4.72490	3
58	0.20734	0.97827	0.21195	4.71813	2
59	0.20763	0.97821	0.21225	4.71137	1
60	0.20791	0.97815	0.21256	4.70463	0
	Cos	Sin	Cot	Tan	Minutes

78 Degrees

12 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.20791	0.97815	0.21256	4.70463	60
1	0.20820	0.97809	0.21286	4.69791	59
2	0.20848	0.97803	0.21316	4.69121	58
3	0.20877	0.97797	0.21347	4.68452	57
4	0.20905	0.97791	0.21377	4.67786	56
5	0.20933	0.97784	0.21408	4.67121	55
6	0.20962	0.97778	0.21438	4.66458	54
7	0.20990	0.97772	0.21469	4.65797	53
8	0.21019	0.97766	0.21499	4.65138	52
9	0.21047	0.97760	0.21529	4.64480	51
10	0.21076	0.97754	0.21560	4.63825	50
11	0.21104	0.97748	0.21590	4.63171	49
12	0.21132	0.97742	0.21621	4.62518	48
13	0.21161	0.97735	0.21651	4.61868	47
14	0.21189	0.97729	0.21682	4.61219	46
15	0.21218	0.97723	0.21712	4.60572	45
16	0.21246	0.97717	0.21743	4.59927	44
17	0.21275	0.97711	0.21773	4.59283	43
18	0.21303	0.97705	0.21804	4.58641	42
19	0.21331	0.97698	0.21834	4.58001	41
20	0.21360	0.97692	0.21864	4.57363	40
21	0.21388	0.97686	0.21895	4.56726	39
22	0.21417	0.97680	0.21925	4.56091	38
23	0.21445	0.97673	0.21956	4.55458	37
24	0.21474	0.97667	0.21986	4.54826	36
25	0.21502	0.97661	0.22017	4.54196	35
26	0.21530	0.97655	0.22047	4.53568	34
27	0.21559	0.97648	0.22078	4.52941	33
28	0.21587	0.97642	0.22108	4.52316	32
29	0.21616	0.97636	0.22139	4.51693	31
30	0.21644	0.97630	0.22169	4.51071	30
31	0.21672	0.97623	0.22200	4.50451	29
32	0.21701	0.97617	0.22231	4.49832	28
33	0.21729	0.97611	0.22261	4.49215	27
34	0.21758	0.97604	0.22292	4.48600	26
35	0.21786	0.97598	0.22322	4.47986	25
36	0.21814	0.97592	0.22353	4.47374	24
37	0.21843	0.97585	0.22383	4.46764	23
38	0.21871	0.97579	0.22414	4.46155	22
39	0.21899	0.97573	0.22444	4.45548	21
40	0.21928	0.97566	0.22475	4.44942	20
41	0.21956	0.97560	0.22505	4.44338	19
42	0.21985	0.97553	0.22536	4.43735	18
43	0.22013	0.97547	0.22567	4.43134	17
44	0.22041	0.97541	0.22597	4.42534	16
45	0.22070	0.97534	0.22628	4.41936	15
46	0.22098	0.97528	0.22658	4.41340	14
47	0.22126	0.97521	0.22689	4.40745	13
48	0.22155	0.97515	0.22719	4.40152	12
49	0.22183	0.97508	0.22750	4.39560	11
50	0.22212	0.97502	0.22781	4.38969	10
51	0.22240	0.97496	0.22811	4.38381	9
52	0.22268	0.97489	0.22842	4.37793	8
53	0.22297	0.97483	0.22872	4.37207	7
54	0.22325	0.97476	0.22903	4.36623	6
55	0.22353	0.97470	0.22934	4.36040	5
56	0.22382	0.97463	0.22964	4.35459	4
57	0.22410	0.97457	0.22995	4.34879	3
58	0.22438	0.97450	0.23026	4.34300	2
59	0.22467	0.97444	0.23056	4.33723	1
60	0.22495	0.97437	0.23087	4.33148	0
	Cos	Sin	Cot	Tan	Minutes

77 Degrees

13 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.22495	0.97437	0.23087	4.33148	60
1	0.22523	0.97430	0.23117	4.32573	59
2	0.22552	0.97424	0.23148	4.32001	58
3	0.22580	0.97417	0.23179	4.31430	57
4	0.22608	0.97411	0.23209	4.30860	56
5	0.22637	0.97404	0.23240	4.30291	55
6	0.22665	0.97398	0.23271	4.29724	54
7	0.22693	0.97391	0.23301	4.29159	53
8	0.22722	0.97384	0.23332	4.28595	52
9	0.22750	0.97378	0.23363	4.28032	51
10	0.22778	0.97371	0.23393	4.27471	50
11	0.22807	0.97365	0.23424	4.26911	49
12	0.22835	0.97358	0.23455	4.26352	48
13	0.22863	0.97351	0.23485	4.25795	47
14	0.22892	0.97345	0.23516	4.25239	46
15	0.22920	0.97338	0.23547	4.24685	45
16	0.22948	0.97331	0.23578	4.24132	44
17	0.22977	0.97325	0.23608	4.23580	43
18	0.23005	0.97318	0.23639	4.23030	42
19	0.23033	0.97311	0.23670	4.22481	41
20	0.23062	0.97304	0.23700	4.21933	40
21	0.23090	0.97298	0.23731	4.21387	39
22	0.23118	0.97291	0.23762	4.20842	38
23	0.23146	0.97284	0.23793	4.20298	37
24	0.23175	0.97278	0.23823	4.19756	36
25	0.23203	0.97271	0.23854	4.19215	35
26	0.23231	0.97264	0.23885	4.18675	34
27	0.23260	0.97257	0.23916	4.18137	33
28	0.23288	0.97251	0.23946	4.17600	32
29	0.23316	0.97244	0.23977	4.17064	31
30	0.23345	0.97237	0.24008	4.16530	30
31	0.23373	0.97230	0.24039	4.15997	29
32	0.23401	0.97223	0.24069	4.15465	28
33	0.23429	0.97217	0.24100	4.14934	27
34	0.23458	0.97210	0.24131	4.14405	26
35	0.23486	0.97203	0.24162	4.13877	25
36	0.23514	0.97196	0.24193	4.13350	24
37	0.23542	0.97189	0.24223	4.12825	23
38	0.23571	0.97182	0.24254	4.12301	22
39	0.23599	0.97176	0.24285	4.11778	21
40	0.23627	0.97169	0.24316	4.11256	20
41	0.23656	0.97162	0.24347	4.10736	19
42	0.23684	0.97155	0.24377	4.10216	18
43	0.23712	0.97148	0.24408	4.09699	17
44	0.23740	0.97141	0.24439	4.09182	16
45	0.23769	0.97134	0.24470	4.08666	15
46	0.23797	0.97127	0.24501	4.08152	14
47	0.23825	0.97120	0.24532	4.07639	13
48	0.23853	0.97113	0.24562	4.07127	12
49	0.23882	0.97106	0.24593	4.06616	11
50	0.23910	0.97100	0.24624	4.06107	10
51	0.23938	0.97093	0.24655	4.05599	9
52	0.23966	0.97086	0.24686	4.05092	8
53	0.23995	0.97079	0.24717	4.04586	7
54	0.24023	0.97072	0.24747	4.04081	6
55	0.24051	0.97065	0.24778	4.03578	5
56	0.24079	0.97058	0.24809	4.03076	4
57	0.24108	0.97051	0.24840	4.02574	3
58	0.24136	0.97044	0.24871	4.02074	2
59	0.24164	0.97037	0.24902	4.01576	1
60	0.24192	0.97030	0.24933	4.01078	0
	Cos	Sin	Cot	Tan	Minutes

76 Degrees

	14 Degree				
Minutes	Sin	Cos	Tan	Cot	
0	0.24192	0.97030	0.24933	4.01078	60
1	0.24220	0.97023	0.24964	4.00582	59
2	0.24249	0.97015	0.24995	4.00086	58
3	0.24277	0.97008	0.25026	3.99592	57
4	0.24305	0.97001	0.25056	3.99099	56
5	0.24333	0.96994	0.25087	3.98607	55
6	0.24362	0.96987	0.25118	3.98117	54
7	0.24390	0.96980	0.25149	3.97627	53
8	0.24418	0.96973	0.25180	3.97139	52
9	0.24446	0.96966	0.25211	3.96651	51
10	0.24474	0.96959	0.25242	3.96165	50
11	0.24503	0.96952	0.25273	3.95680	49
12	0.24531	0.96945	0.25304	3.95196	48
13	0.24559	0.96937	0.25335	3.94713	47
14	0.24587	0.96930	0.25366	3.94232	46
15	0.24615	0.96923	0.25397	3.93751	45
16	0.24644	0.96916	0.25428	3.93271	44
17	0.24672	0.96909	0.25459	3.92793	43
18	0.24700	0.96902	0.25490	3.92316	42
19	0.24728	0.96894	0.25521	3.91839	41
20	0.24756	0.96887	0.25552	3.91364	40
21	0.24784	0.96880	0.25583	3.90890	39
22	0.24813	0.96873	0.25614	3.90417	38
23	0.24841	0.96866	0.25645	3.89945	37
24	0.24869	0.96858	0.25676	3.89474	36
25	0.24897	0.96851	0.25707	3.89004	35
26	0.24925	0.96844	0.25738	3.88536	34
27	0.24954	0.96837	0.25769	3.88068	33
28	0.24982	0.96829	0.25800	3.87601	32
29	0.25010	0.96822	0.25831	3.87136	31
30	0.25038	0.96815	0.25862	3.86671	30
31	0.25066	0.96807	0.25893	3.86208	29
32	0.25094	0.96800	0.25924	3.85745	28
33	0.25122	0.96793	0.25955	3.85284	27
34	0.25151	0.96786	0.25986	3.84824	26
35	0.25179	0.96778	0.26017	3.84364	25
36	0.25207	0.96771	0.26048	3.83906	24
37	0.25235	0.96764	0.26079	3.83449	23
38	0.25263	0.96756	0.26110	3.82992	22
39	0.25291	0.96749	0.26141	3.82537	21
40	0.25320	0.96742	0.26172	3.82083	20
41	0.25348	0.96734	0.26203	3.81630	19
42	0.25376	0.96727	0.26235	3.81177	18
43	0.25404	0.96719	0.26266	3.80726	17
44	0.25432	0.96712	0.26297	3.80276	16
45	0.25460	0.96705	0.26328	3.79827	15
46	0.25488	0.96697	0.26359	3.79378	14
47	0.25516	0.96690	0.26390	3.78931	13
48	0.25545	0.96682	0.26421	3.78485	12
49	0.25573	0.96675	0.26452	3.78040	11
50	0.25601	0.96667	0.26483	3.77595	10
51	0.25629	0.96660	0.26515	3.77152	9
52	0.25657	0.96653	0.26546	3.76709	8
53	0.25685	0.96645	0.26577	3.76268	7
54	0.25713	0.96638	0.26608	3.75828	6
55	0.25741	0.96630	0.26639	3.75388	5
56	0.25769	0.96623	0.26670	3.74950	4
57	0.25798	0.96615	0.26701	3.74512	3
58	0.25826	0.96608	0.26733	3.74075	2
59	0.25854	0.96600	0.26764	3.73640	1
60	0.25882	0.96593	0.26795	3.73205	0
	Cos	Sin	Cot	Tan	Minutes
			75 Degrees		

15 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.25882	0.96593	0.26795	3.73205	60
1	0.25910	0.96585	0.26826	3.72771	59
2	0.25938	0.96578	0.26857	3.72338	58
3	0.25966	0.96570	0.26888	3.71907	57
4	0.25994	0.96562	0.26920	3.71476	56
5	0.26022	0.96555	0.26951	3.71046	55
6	0.26050	0.96547	0.26982	3.70616	54
7	0.26079	0.96540	0.27013	3.70188	53
8	0.26107	0.96532	0.27044	3.69761	52
9	0.26135	0.96524	0.27076	3.69335	51
10	0.26163	0.96517	0.27107	3.68909	50
11	0.26191	0.96509	0.27138	3.68485	49
12	0.26219	0.96502	0.27169	3.68061	48
13	0.26247	0.96494	0.27201	3.67638	47
14	0.26275	0.96486	0.27232	3.67217	46
15	0.26303	0.96479	0.27263	3.66796	45
16	0.26331	0.96471	0.27294	3.66376	44
17	0.26359	0.96463	0.27326	3.65957	43
18	0.26387	0.96456	0.27357	3.65538	42
19	0.26415	0.96448	0.27388	3.65121	41
20	0.26443	0.96440	0.27419	3.64705	40
21	0.26471	0.96433	0.27451	3.64289	39
22	0.26500	0.96425	0.27482	3.63874	38
23	0.26528	0.96417	0.27513	3.63461	37
24	0.26556	0.96410	0.27545	3.63048	36
25	0.26584	0.96402	0.27576	3.62636	35
26	0.26612	0.96394	0.27607	3.62224	34
27	0.26640	0.96386	0.27638	3.61814	33
28	0.26668	0.96379	0.27670	3.61405	32
29	0.26696	0.96371	0.27701	3.60996	31
30	0.26724	0.96363	0.27732	3.60588	30
31	0.26752	0.96355	0.27764	3.60181	29
32	0.26780	0.96347	0.27795	3.59775	28
33	0.26808	0.96340	0.27826	3.59370	27
34	0.26836	0.96332	0.27858	3.58966	26
35	0.26864	0.96324	0.27889	3.58562	25
36	0.26892	0.96316	0.27921	3.58160	24
37	0.26920	0.96308	0.27952	3.57758	23
38	0.26948	0.96301	0.27983	3.57357	22
39	0.26976	0.96293	0.28015	3.56957	21
40	0.27004	0.96285	0.28046	3.56557	20
41	0.27032	0.96277	0.28077	3.56159	19
42	0.27060	0.96269	0.28109	3.55761	18
43	0.27088	0.96261	0.28140	3.55364	17
44	0.27116	0.96253	0.28172	3.54968	16
45	0.27144	0.96246	0.28203	3.54573	15
46	0.27172	0.96238	0.28234	3.54179	14
47	0.27200	0.96230	0.28266	3.53785	13
48	0.27228	0.96222	0.28297	3.53393	12
49	0.27256	0.96214	0.28329	3.53001	11
50	0.27284	0.96206	0.28360	3.52609	10
51	0.27312	0.96198	0.28391	3.52219	9
52	0.27340	0.96190	0.28423	3.51829	8
53	0.27368	0.96182	0.28454	3.51441	7
54	0.27396	0.96174	0.28486	3.51053	6
55	0.27424	0.96166	0.28517	3.50666	5
56	0.27452	0.96158	0.28549	3.50279	4
57	0.27480	0.96150	0.28580	3.49894	3
58	0.27508	0.96142	0.28612	3.49509	2
59	0.27536	0.96134	0.28643	3.49125	1
60	0.27564	0.96126	0.28675	3.48741	0
	Cos	Sin	Cot	Tan	Minutes

74 Degrees

16 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.27564	0.96126	0.28675	3.48741	60
1	0.27592	0.96118	0.28706	3.48359	59
2	0.27620	0.96110	0.28738	3.47977	58
3	0.27648	0.96102	0.28769	3.47596	57
4	0.27676	0.96094	0.28801	3.47216	56
5	0.27704	0.96086	0.28832	3.46837	55
6	0.27731	0.96078	0.28864	3.46458	54
7	0.27759	0.96070	0.28895	3.46080	53
8	0.27787	0.96062	0.28927	3.45703	52
9	0.27815	0.96054	0.28958	3.45327	51
10	0.27843	0.96046	0.28990	3.44951	50
11	0.27871	0.96037	0.29021	3.44576	49
12	0.27899	0.96029	0.29053	3.44202	48
13	0.27927	0.96021	0.29084	3.43829	47
14	0.27955	0.96013	0.29116	3.43456	46
15	0.27983	0.96005	0.29147	3.43084	45
16	0.28011	0.95997	0.29179	3.42713	44
17	0.28039	0.95989	0.29210	3.42343	43
18	0.28067	0.95981	0.29242	3.41973	42
19	0.28095	0.95972	0.29274	3.41604	41
20	0.28123	0.95964	0.29305	3.41236	40
21	0.28150	0.95956	0.29337	3.40869	39
22	0.28178	0.95948	0.29368	3.40502	38
23	0.28206	0.95940	0.29400	3.40136	37
24	0.28234	0.95931	0.29432	3.39771	36
25	0.28262	0.95923	0.29463	3.39406	35
26	0.28290	0.95915	0.29495	3.39042	34
27	0.28318	0.95907	0.29526	3.38679	33
28	0.28346	0.95898	0.29558	3.38317	32
29	0.28374	0.95890	0.29590	3.37955	31
30	0.28402	0.95882	0.29621	3.37594	30
31	0.28429	0.95874	0.29653	3.37234	29
32	0.28457	0.95865	0.29685	3.36875	28
33	0.28485	0.95857	0.29716	3.36516	27
34	0.28513	0.95849	0.29748	3.36158	26
35	0.28541	0.95841	0.29780	3.35800	25
36	0.28569	0.95832	0.29811	3.35443	24
37	0.28597	0.95824	0.29843	3.35087	23
38	0.28625	0.95816	0.29875	3.34732	22
39	0.28652	0.95807	0.29906	3.34377	21
40	0.28680	0.95799	0.29938	3.34023	20
41	0.28708	0.95791	0.29970	3.33670	19
42	0.28736	0.95782	0.30001	3.33317	18
43	0.28764	0.95774	0.30033	3.32965	17
44	0.28792	0.95766	0.30065	3.32614	16
45	0.28820	0.95757	0.30097	3.32264	15
46	0.28847	0.95749	0.30128	3.31914	14
47	0.28875	0.95740	0.30160	3.31565	13
48	0.28903	0.95732	0.30192	3.31216	12
49	0.28931	0.95724	0.30224	3.30868	11
50	0.28959	0.95715	0.30255	3.30521	10
51	0.28987	0.95707	0.30287	3.30174	9
52	0.29015	0.95698	0.30319	3.29829	8
53	0.29042	0.95690	0.30351	3.29483	7
54	0.29070	0.95681	0.30382	3.29139	6
55	0.29098	0.95673	0.30414	3.28795	5
56	0.29126	0.95664	0.30446	3.28452	4
57	0.29154	0.95656	0.30478	3.28109	3
58	0.29182	0.95647	0.30509	3.27767	2
59	0.29209	0.95639	0.30541	3.27426	1
60	0.29237	0.95630	0.30573	3.27085	0
	Cos	Sin	Cot	Tan	Minutes

73 Degrees

17 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.29237	0.95630	0.30573	3.27085	60
1	0.29265	0.95622	0.30605	3.26745	59
2	0.29293	0.95613	0.30637	3.26406	58
3	0.29321	0.95605	0.30669	3.26067	57
4	0.29348	0.95596	0.30700	3.25729	56
5	0.29376	0.95588	0.30732	3.25392	55
6	0.29404	0.95579	0.30764	3.25055	54
7	0.29432	0.95571	0.30796	3.24719	53
8	0.29460	0.95562	0.30828	3.24383	52
9	0.29487	0.95554	0.30860	3.24049	51
10	0.29515	0.95545	0.30891	3.23714	50
11	0.29543	0.95536	0.30923	3.23381	49
12	0.29571	0.95528	0.30955	3.23048	48
13	0.29599	0.95519	0.30987	3.22715	47
14	0.29626	0.95511	0.31019	3.22384	46
15	0.29654	0.95502	0.31051	3.22053	45
16	0.29682	0.95493	0.31083	3.21722	44
17	0.29710	0.95485	0.31115	3.21392	43
18	0.29737	0.95476	0.31147	3.21063	42
19	0.29765	0.95467	0.31178	3.20734	41
20	0.29793	0.95459	0.31210	3.20406	40
21	0.29821	0.95450	0.31242	3.20079	39
22	0.29849	0.95441	0.31274	3.19752	38
23	0.29876	0.95433	0.31306	3.19426	37
24	0.29904	0.95424	0.31338	3.19100	36
25	0.29932	0.95415	0.31370	3.18775	35
26	0.29960	0.95407	0.31402	3.18451	34
27	0.29987	0.95398	0.31434	3.18127	33
28	0.30015	0.95389	0.31466	3.17804	32
29	0.30043	0.95380	0.31498	3.17481	31
30	0.30071	0.95372	0.31530	3.17159	30
31	0.30098	0.95363	0.31562	3.16838	29
32	0.30126	0.95354	0.31594	3.16517	28
33	0.30154	0.95345	0.31626	3.16197	27
34	0.30182	0.95337	0.31658	3.15877	26
35	0.30209	0.95328	0.31690	3.15558	25
36	0.30237	0.95319	0.31722	3.15240	24
37	0.30265	0.95310	0.31754	3.14922	23
38	0.30292	0.95301	0.31786	3.14605	22
39	0.30320	0.95293	0.31818	3.14288	21
40	0.30348	0.95284	0.31850	3.13972	20
41	0.30376	0.95275	0.31882	3.13656	19
42	0.30403	0.95266	0.31914	3.13341	18
43	0.30431	0.95257	0.31946	3.13027	17
44	0.30459	0.95248	0.31978	3.12713	16
45	0.30486	0.95240	0.32010	3.12400	15
46	0.30514	0.95231	0.32042	3.12087	14
47	0.30542	0.95222	0.32074	3.11775	13
48	0.30570	0.95213	0.32106	3.11464	12
49	0.30597	0.95204	0.32139	3.11153	11
50	0.30625	0.95195	0.32171	3.10842	10
51	0.30653	0.95186	0.32203	3.10532	9
52	0.30680	0.95177	0.32235	3.10223	8
53	0.30708	0.95168	0.32267	3.09914	7
54	0.30736	0.95159	0.32299	3.09606	6
55	0.30763	0.95150	0.32331	3.09298	5
56	0.30791	0.95142	0.32363	3.08991	4
57	0.30819	0.95133	0.32396	3.08685	3
58	0.30846	0.95124	0.32428	3.08379	2
59	0.30874	0.95115	0.32460	3.08073	1
60	0.30902	0.95106	0.32492	3.07768	0
	Cos	Sin	Cot	Tan	Minutes

72 Degrees

18 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.30902	0.95106	0.32492	3.07768	60
1	0.30929	0.95097	0.32524	3.07464	59
2	0.30957	0.95088	0.32556	3.07160	58
3	0.30985	0.95079	0.32588	3.06857	57
4	0.31012	0.95070	0.32621	3.06554	56
5	0.31040	0.95061	0.32653	3.06252	55
6	0.31068	0.95052	0.32685	3.05950	54
7	0.31095	0.95043	0.32717	3.05649	53
8	0.31123	0.95033	0.32749	3.05349	52
9	0.31151	0.95024	0.32782	3.05049	51
10	0.31178	0.95015	0.32814	3.04749	50
11	0.31206	0.95006	0.32846	3.04450	49
12	0.31233	0.94997	0.32878	3.04152	48
13	0.31261	0.94988	0.32911	3.03854	47
14	0.31289	0.94979	0.32943	3.03556	46
15	0.31316	0.94970	0.32975	3.03260	45
16	0.31344	0.94961	0.33007	3.02963	44
17	0.31372	0.94952	0.33040	3.02667	43
18	0.31399	0.94943	0.33072	3.02372	42
19	0.31427	0.94933	0.33104	3.02077	41
20	0.31454	0.94924	0.33136	3.01783	40
21	0.31482	0.94915	0.33169	3.01489	39
22	0.31510	0.94906	0.33201	3.01196	38
23	0.31537	0.94897	0.33233	3.00903	37
24	0.31565	0.94888	0.33266	3.00611	36
25	0.31593	0.94878	0.33298	3.00319	35
26	0.31620	0.94869	0.33330	3.00028	34
27	0.31648	0.94860	0.33363	2.99738	33
28	0.31675	0.94851	0.33395	2.99447	32
29	0.31703	0.94842	0.33427	2.99158	31
30	0.31730	0.94832	0.33460	2.98868	30
31	0.31758	0.94823	0.33492	2.98580	29
32	0.31786	0.94814	0.33524	2.98292	28
33	0.31813	0.94805	0.33557	2.98004	27
34	0.31841	0.94795	0.33589	2.97717	26
35	0.31868	0.94786	0.33621	2.97430	25
36	0.31896	0.94777	0.33654	2.97144	24
37	0.31923	0.94768	0.33686	2.96858	23
38	0.31951	0.94758	0.33718	2.96573	22
39	0.31979	0.94749	0.33751	2.96288	21
40	0.32006	0.94740	0.33783	2.96004	20
41	0.32034	0.94730	0.33816	2.95721	19
42	0.32061	0.94721	0.33848	2.95437	18
43	0.32089	0.94712	0.33881	2.95155	17
44	0.32116	0.94702	0.33913	2.94872	16
45	0.32144	0.94693	0.33945	2.94591	15
46	0.32171	0.94684	0.33978	2.94309	14
47	0.32199	0.94674	0.34010	2.94028	13
48	0.32227	0.94665	0.34043	2.93748	12
49	0.32254	0.94656	0.34075	2.93468	11
50	0.32282	0.94646	0.34108	2.93189	10
51	0.32309	0.94637	0.34140	2.92910	9
52	0.32337	0.94627	0.34173	2.92632	8
53	0.32364	0.94618	0.34205	2.92354	7
54	0.32392	0.94609	0.34238	2.92076	6
55	0.32419	0.94599	0.34270	2.91799	5
56	0.32447	0.94590	0.34303	2.91523	4
57	0.32474	0.94580	0.34335	2.91246	3
58	0.32502	0.94571	0.34368	2.90971	2
59	0.32529	0.94561	0.34400	2.90696	1
60	0.32557	0.94552	0.34433	2.90421	0
	Cos	Sin	Cot	Tan	Minutes

71 Degrees

	19 Degree				
Minutes	Sin	Cos	Tan	Cot	
0	0.32557	0.94552	0.34433	2.90421	60
1	0.32584	0.94542	0.34465	2.90147	59
2	0.32612	0.94533	0.34498	2.89873	58
3	0.32639	0.94523	0.34530	2.89600	57
4	0.32667	0.94514	0.34563	2.89327	56
5	0.32694	0.94504	0.34596	2.89055	55
6	0.32722	0.94495	0.34628	2.88783	54
7	0.32749	0.94485	0.34661	2.88511	53
8	0.32777	0.94476	0.34693	2.88240	52
9	0.32804	0.94466	0.34726	2.87970	51
10	0.32832	0.94457	0.34758	2.87700	50
11	0.32859	0.94447	0.34791	2.87430	49
12	0.32887	0.94438	0.34824	2.87161	48
13	0.32914	0.94428	0.34856	2.86892	47
14	0.32942	0.94418	0.34889	2.86624	46
15	0.32969	0.94409	0.34922	2.86356	45
16	0.32937	0.94399	0.34954	2.86089	44
17	0.33024	0.94390	0.34987	2.85822	43
18	0.33051	0.94380	0.35020	2.85555	42
19	0.33079	0.94370	0.35052	2.85289	41
20	0.33106	0.94361	0.35085	2.85023	40
21	0.33134	0.94351	0.35118	2.84758	39
22	0.33161	0.94342	0.35150	2.84494	38
23	0.33189	0.94332	0.35183	2.84229	37
24	0.33216	0.94322	0.35216	2.83965	36
25	0.33244	0.94313	0.35248	2.83702	35
26	0.33271	0.94303	0.35281	2.83439	34
27	0.33298	0.94293	0.35314	2.83176	33
28	0.33326	0.94284	0.35346	2.82914	32
29	0.33353	0.94274	0.35379	2.82653	31
30	0.33381	0.94264	0.35412	2.82391	30
31	0.33408	0.94254	0.35445	2.82130	29
32	0.33436	0.94245	0.35477	2.81870	28
33	0.33463	0.94235	0.35510	2.81610	27
34	0.33490	0.94225	0.35543	2.81350	26
35	0.33518	0.94215	0.35576	2.81091	25
36	0.33545	0.94206	0.35608	2.80833	24
37	0.33573	0.94196	0.35641	2.80574	23
38	0.33600	0.94186	0.35674	2.80316	22
39	0.33627	0.94176	0.35707	2.80059	21
40	0.33655	0.94167	0.35740	2.79802	20
41	0.33682	0.94157	0.35772	2.79545	19
42	0.33710	0.94147	0.35805	2.79289	18
43	0.33737	0.94137	0.35838	2.79033	17
44	0.33764	0.94127	0.35871	2.78778	16
45	0.33792	0.94118	0.35904	2.78523	15
46	0.33819	0.94108	0.35937	2.78269	14
47	0.33846	0.94098	0.35969	2.78014	13
48	0.33874	0.94088	0.36002	2.77761	12
49	0.33901	0.94078	0.36035	2.77507	11
50	0.33929	0.94068	0.36068	2.77254	10
51	0.33956	0.94058	0.36101	2.77002	9
52	0.33983	0.94049	0.36134	2.76750	8
53	0.34011	0.94039	0.36167	2.76498	7
54	0.34038	0.94029	0.36199	2.76247	6
55	0.34065	0.94019	0.36232	2.75996	5
56	0.34093	0.94009	0.36265	2.75746	4
57	0.34120	0.93999	0.36298	2.75496	3
58	0.34147	0.93989	0.36331	2.75246	2
59	0.34175	0.93979	0.36364	2.74997	1
60	0.34202	0.93969	0.36397	2.74748	0
	Cos	Sin	Cot	Tan	Minutes
			70 Degrees		

20 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.34202	0.93969	0.36397	2.74748	60
1	0.34229	0.93959	0.36430	2.74499	59
2	0.34257	0.93949	0.36463	2.74251	58
3	0.34284	0.93939	0.36496	2.74004	57
4	0.34311	0.93929	0.36529	2.73756	56
5	0.34339	0.93919	0.36562	2.73509	55
6	0.34366	0.93909	0.36595	2.73263	54
7	0.34393	0.93899	0.36628	2.73017	53
8	0.34421	0.93889	0.36661	2.72771	52
9	0.34448	0.93879	0.36694	2.72526	51
10	0.34475	0.93869	0.36727	2.72281	50
11	0.34503	0.93859	0.36760	2.72036	49
12	0.34530	0.93849	0.36793	2.71792	48
13	0.34557	0.93839	0.36826	2.71548	47
14	0.34584	0.93829	0.36859	2.71305	46
15	0.34612	0.93819	0.36892	2.71062	45
16	0.34639	0.93809	0.36925	2.70819	44
17	0.34666	0.93799	0.36958	2.70577	43
18	0.34694	0.93789	0.36991	2.70335	42
19	0.34721	0.93779	0.37024	2.70094	41
20	0.34748	0.93769	0.37057	2.69853	40
21	0.34775	0.93759	0.37090	2.69612	39
22	0.34803	0.93748	0.37123	2.69371	38
23	0.34830	0.93738	0.37157	2.69131	37
24	0.34857	0.93728	0.37190	2.68892	36
25	0.34884	0.93718	0.37223	2.68653	35
26	0.34912	0.93708	0.37256	2.68414	34
27	0.34939	0.93698	0.37289	2.68175	33
28	0.34966	0.93688	0.37322	2.67937	32
29	0.34993	0.93677	0.37355	2.67700	31
30	0.35021	0.93667	0.37388	2.67462	30
31	0.35048	0.93657	0.37422	2.67225	29
32	0.35075	0.93647	0.37455	2.66989	28
33	0.35102	0.93637	0.37488	2.66752	27
34	0.35130	0.93626	0.37521	2.66516	26
35	0.35157	0.93616	0.37554	2.66281	25
36	0.35184	0.93606	0.37588	2.66046	24
37	0.35211	0.93596	0.37621	2.65811	23
38	0.35239	0.93585	0.37654	2.65576	22
39	0.35266	0.93575	0.37687	2.65342	21
40	0.35293	0.93565	0.37720	2.65109	20
41	0.35320	0.93555	0.37754	2.64875	19
42	0.35347	0.93544	0.37787	2.64642	18
43	0.35375	0.93534	0.37820	2.64410	17
44	0.35402	0.93524	0.37853	2.64177	16
45	0.35429	0.93514	0.37887	2.63945	15
46	0.35456	0.93503	0.37920	2.63714	14
47	0.35484	0.93493	0.37953	2.63483	13
48	0.35511	0.93483	0.37986	2.63252	12
49	0.35538	0.93472	0.38020	2.63021	11
50	0.35565	0.93462	0.38053	2.62791	10
51	0.35592	0.93452	0.38086	2.62561	9
52	0.35619	0.93441	0.38120	2.62332	8
53	0.35647	0.93431	0.38153	2.62103	7
54	0.35674	0.93420	0.38186	2.61874	6
55	0.35701	0.93410	0.38220	2.61646	5
56	0.35728	0.93400	0.38253	2.61418	4
57	0.35755	0.93389	0.38286	2.61190	3
58	0.35782	0.93379	0.38320	2.60963	2
59	0.35810	0.93368	0.38353	2.60736	1
60	0.35837	0.93358	0.38386	2.60509	0
	Cos	Sin	Cot	Tan	Minutes

69 Degrees

C-37 EN0591

ERROR: syntaxerror
OFFENDING COMMAND: --nostringval--

STACK: